

CFD INVESTIGATION OF MULTI-SPECIES SONIC JET MIXING IN SUPERSONIC CROSSFLOW

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Abstract—The mixing process of a fuel jet in a supersonic crossflow remains a critical challenge in the design of scramjet combustors. This study employs orthogonal analysis to systematically investigate the influence of supersonic mainstream and fuel jet parameters on the mixing process. The simulated equations are three-dimensional Reynolds-averaged Navier-Stokes (RANS) equations coupled with the shear stress transport (SST) turbulence model. The ongoing study reveals that jet patterns can be categorized into three distinct types based on the velocity ratio: attachment, transition, and separation patterns. These relations offer valuable insights for the preliminary optimization of supersonic combustors.

This research is ongoing, with further investigations aimed at exploring their applicability to a broader range of supersonic combustion scenarios. The findings are expected to contribute significantly to the advancement of scramjet combustor design and performance optimization based on difference analysis, which indicates that the $p_{t,0}$, $p_{t,f}$ and Ma_0 have a remarkable impact on the Pressure, Temperature, and Velocity.

I. INTRODUCTION

A. Background

Supersonic mixing and combustion are key issues for the scramjet propulsion systems, thus they have attracted attention since 1958. Supersonic combustion is a necessary condition for a scramjet to realize hypersonic flight. It mainly involves some fundamental issues, such as ignition process, flame structure analysis, flame stabilization process, flame propagation process, and so on[1]. However, before the discussion of supersonic combustion, there is an acknowledged challenge for sufficient mixing between the supersonic mainstream and the fuel due to the short residence time of the mainstream in the scramjet combustor, and it is on the order of milliseconds for typical flight conditions. Because of its simple structure and good mixing performance for scramjet[2].

The transverse fuel injection through a wall orifice has been proven to be an efficient fuel supply strategy for the scramjet engine. The mixing characteristics of a fuel jet into a supersonic mainstream are a fundamental issue that initially drew the interest of many researchers. The three-dimensional structure of the transverse jet near the jet orifice is the most widely and deeply studied feature[3] (see Fig. 1.1).

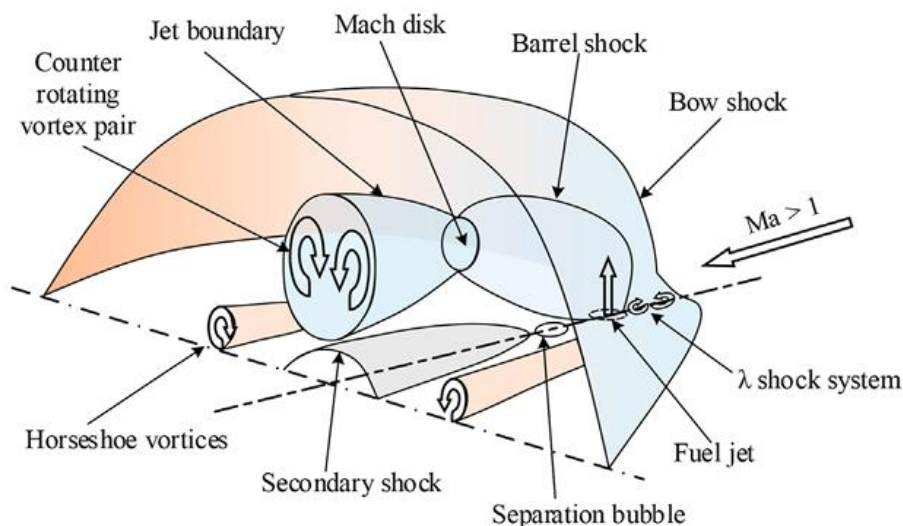


Fig. 1.1 - Schematic diagram of the transverse jet into the supersonic mainstream

B. Theory

The mixing process between the injectant and the supersonic crossflow is an important issue in scramjet engine design, and efficient mixing significantly improves combustion efficiency. A hovering vortex is formed between the separation region and the barrel shock wave, and this may be induced by the large negative density gradient.

GEOMETRIC MODEL AND CASE SETTING

The computational domain of the supersonic combustor is presented in Fig. 1.2. The domain is a rectangular configuration, and the width, height, and length of the entrance were 100 mm, 100 mm, and 900 mm, respectively. The origin of the coordinate system was set at the center of the jet hole. The center of the jet hole was 100 mm downstream of the entrance, and its diameter is a variable. The walls of the configuration in the study were assumed to be no-slip and adiabatic, which was the same as the verified cases mentioned above. The grid system of the computational domain is structured as Fig. 1.3 of Case 1 as an example. It shows that the mesh near the jet hole and wall is much denser to enhance the accuracy of the numerical results.

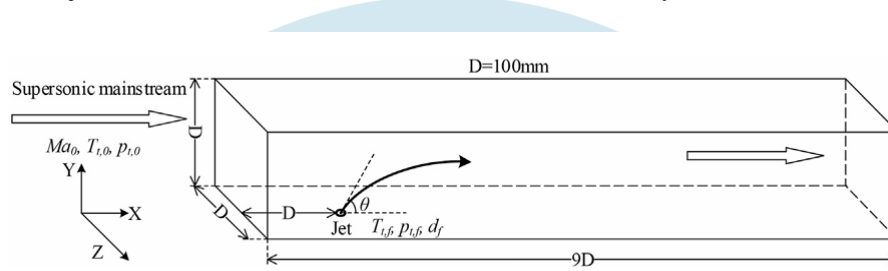


Fig. 1.2 Schematic computational domain of the supersonic combustor

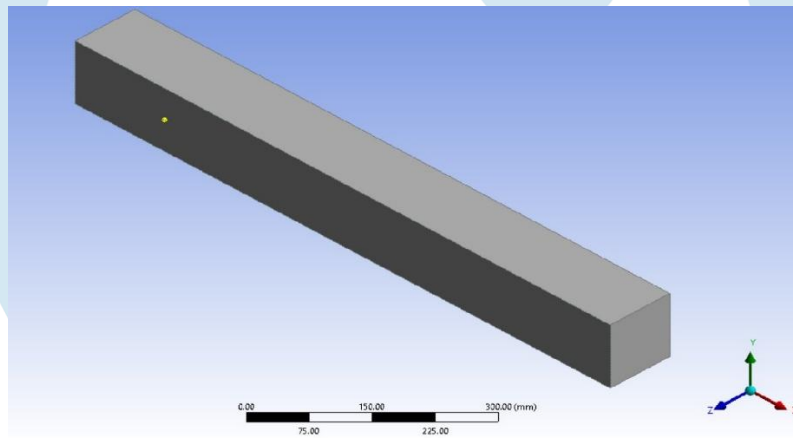


Fig. 1.3 Schematic geometry of the Supersonic Combustor.

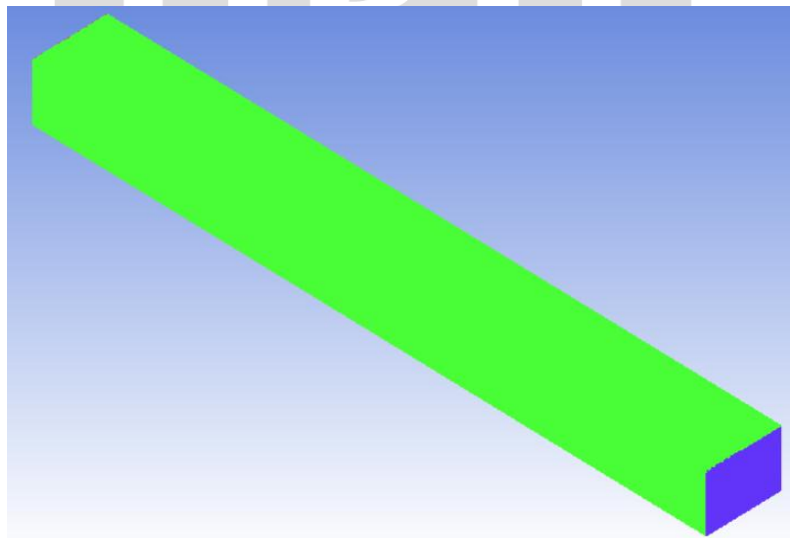


Fig. 1.4 Mesh of the Supersonic Combustor.

Mesh Data:

Minimum Orthogonal Quality = 5.11648e-01

Maximum Aspect Ratio = 7.73574e+01

Nodes: 16546872 nodes

Total elements: 16814641

Cells: 16276615 hexahedral cells

Element types :

Line_2: 76

Hexa_8: 16276615

Quad_4: 537950

Element parts :

Air Inlet: 16303

Fuel Inlet: 361

Geometry: 76

Outlet: 16303
 Solid: 16276615
 Wall: 504983

The orthogonal method was used to carry out the present numerical study. The pre-determined parameters for the supersonic mainstream included the Mach number Ma_0 , total temperature $T_{t,0}$, and total pressure $P_{t,0}$, correspondingly. The parameter variables for the jet hole were total temperature $T_{t,f}$, total pressure, hole diameter d_f , jet angle θ , and the type of working fluid, respectively.

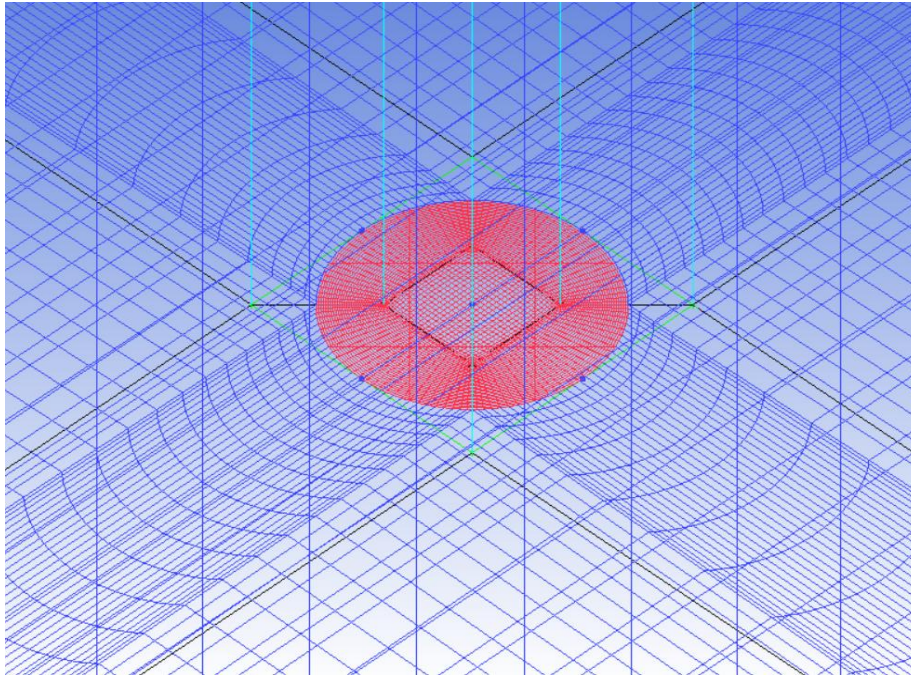


Fig. 1.5 Fuel Inlet of the Supersonic Combustor.

II. LITERATURE SURVEY

This study investigates the influences of the parameters of the supersonic mainstream and the fuel jet on the mixing process. This was performed by the three-dimensional Reynolds-averaged Navier-Stokes (RANS) coupled with the shear stress transport (SST) turbulence model. The results show that the jet patterns can be divided into three categories by calculating the velocity ratio, named attachment pattern, transition pattern, and separation pattern, respectively. The extreme difference analysis indicates that the total pressure and Mach number of the mainstream, the total pressure of the fuel jet, have a remarkable impact on the penetration depth and total pressure recovery. And a positive correlation between the penetration depth and mixing efficiency has been elaborated based on analyzing the influence mechanism of the combustion process.

Zhao J, Lin W, Yan C, Zheng Z, Tong Y, Nie W. The results of experiment and simulation show that using H_2 -Water combined jets, the penetration depth of the jet spray can be greatly increased and the jet mixing effect can be significantly improved, which will contribute to the engine's ignition and stable combustion. In the case of pre-water/post- H_2 , the penetration depth of the hydrogen jet is greater. In the case of pre- H_2 /post-water, the hydrogen jet raises the water spray mainly by protecting the integrity of the water column[1].

Du Z, Shen C, Huang W, Han Y. The mixing speed and the mixing efficiency both increase with the increase of the intensity of the shock wave. The mixing enhancement mechanism is that the enhancement of the streamwise and spanwise vorticity upstream of the jet promotes the enlargement of the upstream separation zone and recirculation zone. When the shock wave position moves downstream, the spanwise vorticity increases, and the downstream recirculation zone increases significantly to promote mixing[2].

Du Z-b, Shen C-b, Huang W, Zhong X-y. Results of the three-dimensional Reynolds-average Navier-Stokes (RANS) equations coupled with the two-equation shear stress transport (SST) κ - ω turbulence model show that the combined strategy of the oblique shock wave and secondary recirculation jet device can effectively improve the mixing speed and mixing efficiency with little total pressure loss. Also, the secondary recirculation jet device can reduce the peak of the heat flux effectively. In this study, the case with the single bleed hole owns the best effect with improving the mixing efficiency by 82.75% locally and reducing the maximum heat flux by 15.24% respectively[3].

Yuelei Zhang, Puneet Rana, R. Moradi, Zhixiong Li, has examined to obtain an optimum jet arrangement in the combustor chamber. Our study shows that the injection of the coaxial air/hydrogen jet noticeably improves mixing downstream by augmentation of fuel interaction with an air jet. Our results also show that fuel jet space of 7 Dj offers maximum fuel mixing by the formation of multi vortices with uniform strength[4].

Miaomiao WANG, Ziniu WU have considered, five moving triple points, with each connecting an incident shock wave, a reflected shock wave and a Mach stem, are identified. By using the reference frame co-moving with each triple point, the type of each shock wave of this triple point is clarified. The present study is significant in that it treats a new shock reflection problem leading to a new shock reflection configuration and showing potential applications in supersonic flow with unsteady shock interaction[5].

Gautam Choubey, Devarajan Yuvarajan, Ahmad Shafee, K.M. Pandey. This review reveals that the multi-porthole injection strategy is favorable in accelerating the wall cooling and near-field mixing along with reducing the loss of total pressure whereas a parametric investigation has a promising effect on jet penetration as well as mixing improvement in supersonic cross-flow. Additionally, the formation of the shock upstream of the fuel jet is greatly helpful in the mixing process in the scramjet engines, hence sinusoidal as well as a wavy wall are used in transverse injection flow-field[6].

G Choubey, D Yuvarajan, W Huang, L Yan, H Babazadeh, KM Pandey have promoted hydrogen as a potential fuel in scramjet engines. As H_2 is also environmentally safe and clean and can be produced from abundant sources, several researchers across the globe support the use of H_2 fuel. However, its lower volumetric energy density and higher flammability range also have an adverse effect in on-board storage system for aircraft propulsion applications. This review gives a brief representation of Hydrogen fueled scramjet engine as well the challenges associated with H_2 fuel[7].

M Sun, H Wang, Z Cai, J Zhu focused on Science and Technology on Scramjet Laboratory at National University of Defense Technology has carried out a huge amount of studies on fuel injection, mixing and combustion in Scramjet combustors, which significantly promotes the development of Scramjet engines in China. This book summarizes the research on unsteady supersonic combustion that has been carried out by our group in the past 15 years, and presents many state-of-the-art results and analyses in this subject. The book is aimed at graduate students majoring in aeronautical and aerospace engineering, as well as researchers and engineers working in design of Scramjet engines. The prerequisite knowledge includes fluid mechanics, combustion principles, and computational fluid dynamics[8].

Vatsalya Sharma, Vinayak Eswaran, Debasis Chakraborty have observed that in every case positive non-zero angles of injection, in the direction of the crossflow, increase thermodynamic efficiency, while the negative non-zero angles, opposing the crossflow, augment mixing. As mixing is of paramount importance in the SCRAMJET engine, due to high speeds and low residence times, we conclude that the best option is to have the angle of fuel jet injection in the direction opposing the incoming flow. This recommendation has not been seen yet in the research literature. The degree to which the infusion is slanted towards the incoming flow can be decided based on the desired rate of the simultaneous penetration of the fuel into the recirculating flame-holder, which increases with increasing angle[9].

Tim Roos, Adrian Pudsey, Mathew Bricalli, Hideaki Ogawa have found out that Wall heat flux increases in configurations with cavities, particularly on the aft wall of the cavity, while fuel drawn into the cavity is seen to contribute to wall cooling in case of high wall temperatures. This can reduce wall cooling requirements and simplify combustor design. In general, the enhanced mixing and jet penetration induced by the cavity could allow for shorter combustor designs, which in turn allows for more compact flight vehicle design[10].

Majie Zhao, Yifan Bian, Qinling Li, Taohong Ye have got the results of the two-dimensional and three-dimensional streamlines illustrate that the trailing counter-rotating vortex pairs (TCVP), the secondary TCVP of primary jet and the horseshoe vortex can merge and form a new horseshoe vortex. Three counter-rotating vortex pairs (CVP) are formed in the downstream of secondary jet: the CVP-B due to interactions between the supersonic crossflow and secondary jet; the CVP-C due to interactions between the supersonic crossflow and the CVP-D due to interactions of the supersonic crossflow and primary jet[11].

Zun Cai, Jiajian Zhu, Xue-Song Bai have established a stable flame in the shear layer between the downstream part of the cavity and the outer supersonic flow. It is concluded that the ignition processes excited by the LIP can be divided into a LIP initiation regime and a transient ignition reaction regime. Both the fueling rate and the LIP energy significantly affect the cavity ignition processes. Increasing the fueling rate or the laser energy can shorten the ignition processes in the cavity. A weak ignition mode and an intense ignition mode are postulated to explain the combustion behavior of the ignition processes in the cavity-based supersonic combustor[12].

Cai Xiaodong, Liang Jianhan, Ralf Deiterding, Yasser Mahmoudi, Sun Mingbo provided the numerical results of subsonic combustion near the walls induced by the boundary layers, the OSIC/MSID is not entirely symmetrical, while for the pure OSIC mode, larger fluctuations are observed along the oblique shock waves resulting from enhanced instabilities due to additional chemical heat release[13].

Mingbo Sun and Zhiwei Hu Analysis on streamlines passing the separation region shows that the wing of the herringbone separation bubble serves as a micro-ramp vortex generator and streamlines acquire angular momentum downstream to form a secondary surface TCVP in the reattachment valley. Herringbone separation wings disappear in the far field due to the cross-interaction of lateral supersonic flow and the expansion flow in the reattachment valley, which also leads to the vanishing of the secondary TCVP. A three-dimensional schematic of surface trailing wakes is presented and explains the formation mechanisms of the surface TCVPs.[14]

Wei Huang, Ming-hui Li, Li Yan have implied that the intense combustion downstream of the injector can enhance the mixing process between the injectant and air, and the mixing and combustion process can be enhanced mutually. When the pseudo shock wave is pushed upstream of the wall orifice, more injectant is brought into the separation zone upstream of the injector, which is beneficial for the mixing process between the injectant and air[15].

III. METHODOLOGY

For this project, ANSYS software is used, and the project discusses about the analysis of the model, which is carried out in ANSYS Fluent, where the results are extracted with definite boundary conditions.

A. ANSYS Software:

ANSYS is a leading engineering simulation software that enables designers and engineers to analyse and optimise complex product designs across a range of industries, including aerospace, automotive, electronics, energy, and biomedical. Founded in 1970, ANSYS provides a comprehensive suite of tools for various physics simulations, such as structural mechanics, fluid dynamics (CFD), electromagnetics, heat transfer, and even Multiphysics simulations, where several types of physical phenomena interact.

One of the key features of ANSYS software is its ability to handle complex geometries and large-scale simulations with high accuracy. The software's advanced meshing tools create high-quality computational grids, essential for precise simulations. ANSYS also integrates powerful post-processing capabilities to visualise and interpret results, making it easier to understand and improve designs. Through these advanced features, ANSYS helps companies reduce the need for costly physical prototypes, shorten development cycles, and enhance product performance and reliability.

B. Penetration depth

The extreme difference analysis method is employed to obtain more useful information in this orthogonal work. If the extreme difference is larger, it indicates that the influence of the factor on the objective functions is larger.

Here is another interesting thing: the jet angle θ has a weak influence on the penetration depth, which seems counterintuitive. For supersonic crossflow, the near field crossflow pressure p_2 is that after the bow shock caused by the fuel jet into the supersonic mainstream. (β and θ are the shockwave angle and the flow deflection angle, respectively).

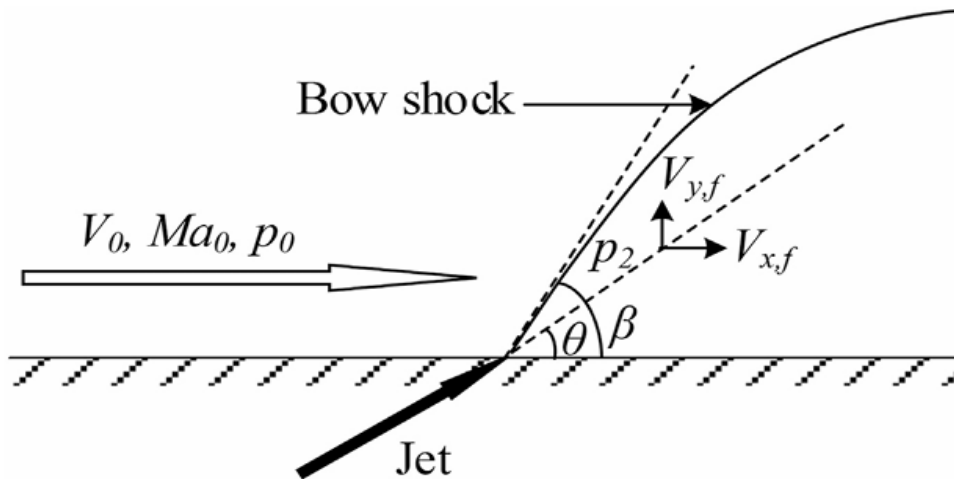


Fig. 3.1-Schematic of the supersonic crossflow.

C. Total Pressure Recovery

The jet angle θ has a negative effect on the total pressure recovery, which is different from that of jet angle on the penetration depth. This is because as the jet angle θ increases, the shock wave angle β will increase together. Its increase will make the intensity of the bow shock approach that of the normal shock, resulting in a greater total pressure loss. Considering the main factors influencing the total pressure recovery are the same as those of the penetration depth.

D. Mixing Efficiency

The mixing process mainly involved mass and momentum exchange between the two fluids (mainstream and fuel jet). Moreover, from a microscopic perspective, the essence of the mixing process is the diffusion degree between the fuel jet and the supersonic mainstream under finite space-time conditions. So now we need to start looking for reasonable variables to represent the diffusion time, the total mass flow rate needed to be diffused, and the diffusion rate in the mixing process. Therefore, a dimensionless number LJ was established from this perspective.

In this paper, the cell-centred finite volume approach was employed to solve the RANS equations along with the density-based double precision. The second-order spatially accurate upwind scheme with advection upstream splitting method (AUSM) flux vector splitting is utilised to accelerate convergence, and the Courant-Friedrichs-Levy (CFL) number is set to 0.5 with proper under-relaxation factors to ensure stability [12].

The governing equations are expressed as follows[12]:

Species transport equation,

$$\frac{\partial(\rho Y_s)}{\partial t} + \frac{\partial(\rho Y_s u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\rho D_s \frac{\partial Y_s}{\partial x_j} \right), \quad s = 1, 2, \dots, ns$$

Continuity equation,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0$$

Energy Equation,

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial(\rho H u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\tau_{ij} u_i + k \frac{\partial T}{\partial x_j} + \sum_{s=1}^{ns} \rho D_s h_s \frac{\partial Y_s}{\partial x_j} \right)$$

Equation of state for an ideal gas,

$$p = \rho R T \sum_{s=1}^{ns} \frac{Y_s}{M_s}$$

Where ρ is density,

u is velocity,

p is pressure,

T is static temperature,

E and H are the total energy and total enthalpy per unit volume,

R is the universal constant of gas,

M_s , Y_s , D_s , and h_s are the molecular weight, τ_{ij} is the molecular stress tensor,

mass fraction, mass diffusion coefficient, and absolute enthalpy per unit mass of species s .

Table 3.1: Values for each design variable employed in the current study, considering Methane, Hydrogen, Butane and Kerosene fuels.

Factors	Flow Parameters
$P_{t,0}/\text{atm}$	5
$T_{t,0}/\text{k}$	1000
$M_{a,0}$	2.0
$P_{f,t}/\text{atm}$	5
$T_{t,t}/\text{atm}$	300

IV. RESULTS AND DISCUSSION

A. General Observations Across All Fuels

The numerical investigation of sonic jet mixing into supersonic crossflows reveals several consistent phenomena across all fuels studied. The **velocity ratio** between the jet and the mainstream determines the jet pattern: attachment, transition, or separation. The **penetration depth** is strongly influenced by mainstream Mach number, total pressure, and jet hole diameter. The **total pressure recovery** is sensitive to shockwave angle (β) and jet angle (θ), with larger angles producing stronger bow shocks and greater losses. The **mixing efficiency** correlates positively with the dimensionless LJ number, which captures diffusion time and vortex-driven mixing.

Across fuels, gaseous injectants (hydrogen, methane, butane) exhibit faster ignition and stronger vortex-driven mixing due to lower density and higher jet velocity. Liquid fuels (kerosene) provide higher energy density but suffer from atomization and evaporation delays. These trade-offs define the practical challenges in scramjet combustor design.

B. Hydrogen Jet in Supersonic Crossflow

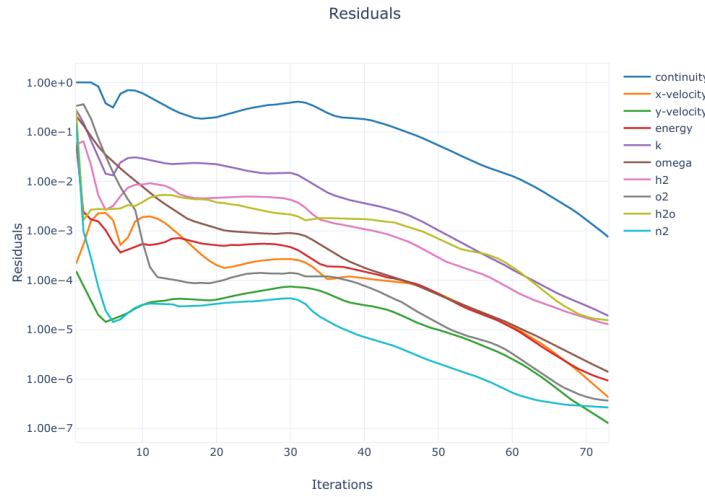


Fig. 4.1: Convergence History of Residuals



Fig. 4.2: Velocity Magnitude Contours (m/s)



Fig. 4.3: Density Contours (kg/m^3)

- Density & Velocity: Lowest density ($3.51 \times 10^9 \text{ kg/m}^3$) and highest velocity ($7.99 \times 10^3 \text{ m/s}$).
- Combustion Characteristics: Extremely fast ignition, high diffusivity, clean combustion.
- Ignition Delay: Very short; ideal for scramjets with millisecond residence times.
- Shock–Fuel Interaction: Weak bow shocks, minimizing pressure losses; strong vortex generation enhances mixing.
- Mixing Efficiency: Highest among all fuels; LJ number strongly correlates with efficiency.
- Penetration Height: Limited due to low density; requires optimization of jet diameter/angle.
- Combustion Stability: Highly stable once ignited; often used as pilot fuel.
- Implications: Benchmark fuel for scramjet research; excellent for ignition studies but limited by storage and volumetric energy density.

C. Kerosene Jet in Supersonic Crossflow

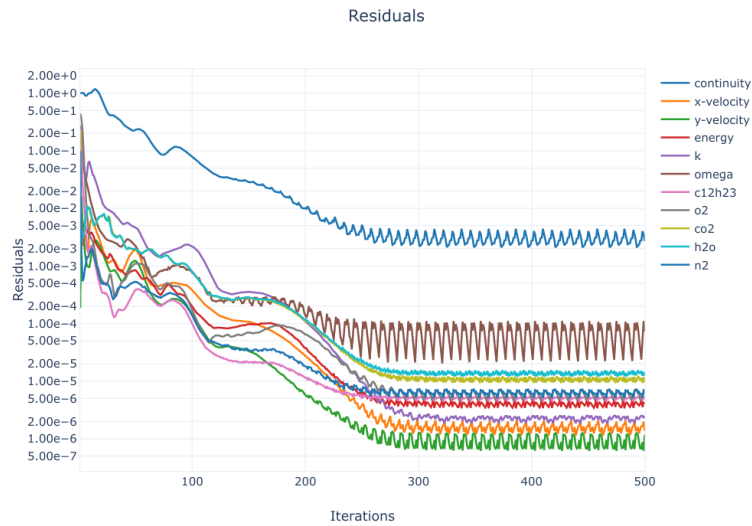


Fig. 4.4: Convergence History of Residuals

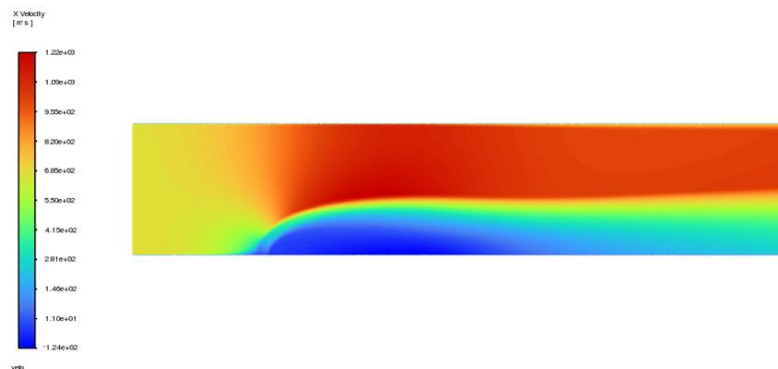
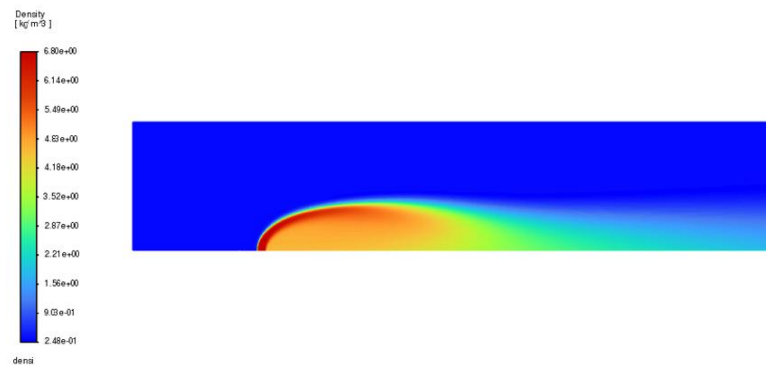


Fig 4.5: Velocity Magnitude Contours (m/s)

Fig. 4.6: Density Contours (kg/m³)

- Density & Velocity: Highest density (6.9×10^0 kg/m³), moderate velocity (1.22×10^3 m/s).
- Combustion Characteristics: High energy density; requires atomization and evaporation.
- Ignition Delay: Long; problematic for short residence times.
- Shock–Fuel Interaction: Strong bow shocks, increasing penetration but causing significant pressure losses.
- Mixing Efficiency: Lowest; delayed evaporation reduces efficiency.
- Penetration Height: Highest among fuels; deep penetration into mainstream.
- Combustion Stability: Less stable in supersonic flows; stability improved with pilot fuels.
- Implications: Practical operational fuel; research focuses on atomization, dual-fuel strategies, and injection optimization.

D. Methane Jet in Supersonic Crossflow

Residuals

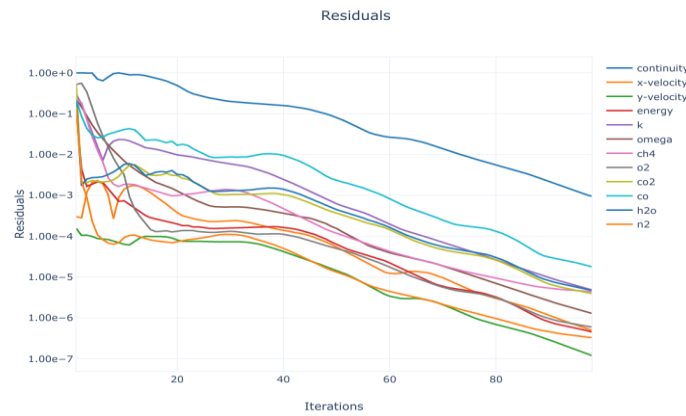


Fig. 4.7: Convergence History of Residuals

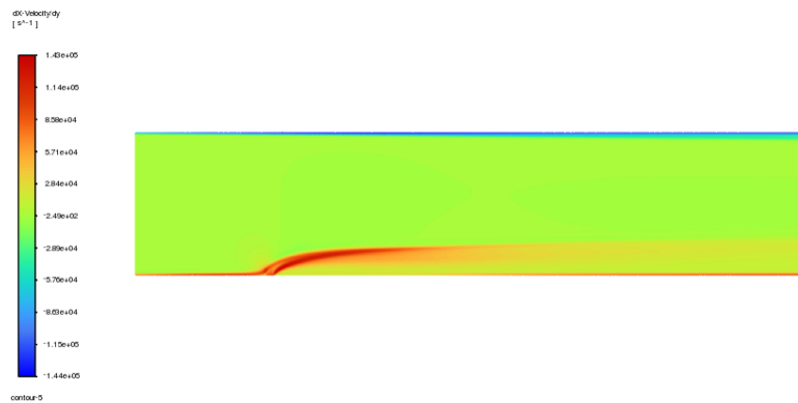


Fig 4.8: Velocity Magnitude Contours (m/s)

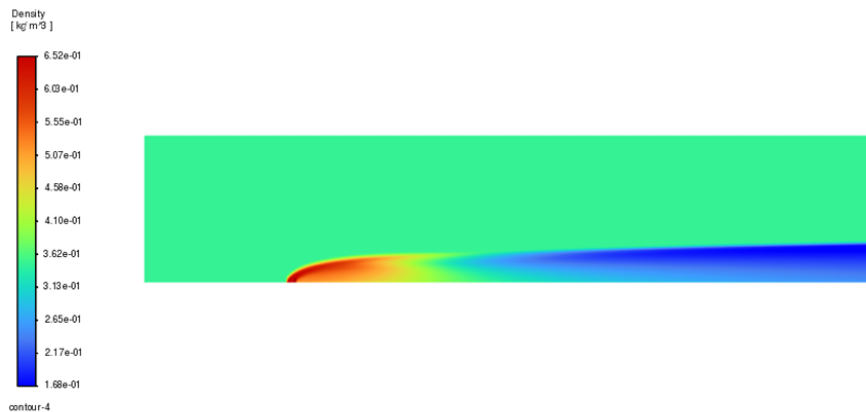


Fig. 4.9: Density Contours (kg/m³)

- Density & Velocity: Moderate density ($6.52 \times 10^0 \text{ kg/m}^3$), velocity ($8.78 \times 10^2 \text{ m/s}$).
- Combustion Characteristics: Cleaner combustion than kerosene; balanced properties.
- Ignition Delay: Moderate; shorter than kerosene, longer than hydrogen.
- Shock–Fuel Interaction: Moderate bow shocks; balanced penetration and pressure recovery.
- Mixing Efficiency: Moderate; LJ correlation positive but weaker than hydrogen.
- Penetration Height: Intermediate; sufficient for combustion without excessive losses.
- Combustion Stability: Moderately stable; benefits from dual-fuel systems.
- Implications: Good compromise fuel; suitable for hybrid strategies and experimental validation.

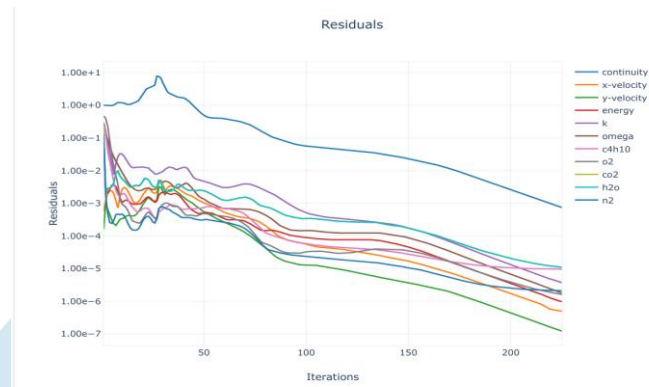
E. Butane Jet in Supersonic Crossflow

Fig. 4.10: Convergence History of Residuals

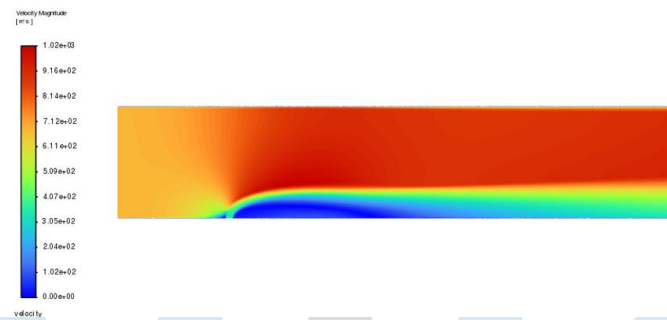
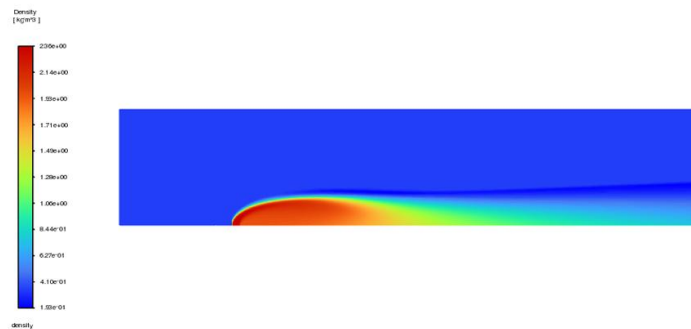


Fig 4.11: Velocity Magnitude Contours (m/s)

Fig. 4.12: Density Contours (kg/m^3)

- Density & Velocity: High density ($2.36 \times 10^{10} \text{ kg/m}^3$), velocity ($1.02 \times 10^3 \text{ m/s}$).
- Combustion Characteristics: High energy density; gaseous nature aids mixing.
- Ignition Delay: Shorter than kerosene, longer than hydrogen.
- Shock–Fuel Interaction: Strong bow shocks; high penetration but pressure recovery penalties.
- Mixing Efficiency: Higher than kerosene, lower than hydrogen.
- Penetration Height: Significant; comparable to kerosene.
- Combustion Stability: Relatively stable; gaseous phase reduces ignition delay.
- Implications: Hybrid candidate fuel; balances penetration and mixing, useful in multi-species injection.

Table 4.1: Comparison of Fuel Characteristics for Supersonic Combustion.

Fuel	Density (kg/m ³)	Velocity (m/s)	Ignition Delay	Penetration Depth	Mixing Efficiency	Stability	Key Implication
Hydrogen	3.51×10^9	7.99×10^3	Very short	Low	Very high	High	Benchmark fuel, pilot ignition
Kerosene	6.9×10^{10}	1.22×10^3	Long	Very high	Low	Low–Moderate	Operational fuel, atomization needed
Methane	6.52×10^9	8.78×10^2	Moderate	Medium	Medium	Moderate	Balanced compromise fuel
Butane	2.36×10^{10}	1.02×10^3	Short–Moderate	High	Medium–High	Moderate–High	Hybrid injection candidate

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