An Extended Penalty Method To Solve Transportation Problem

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ABSRTACT-- The transportation problem aims to find the best way to move goods from suppliers to customers at the lowest possible cost, while making sure all supply and demand needs are fulfilled. There are several methods to solve transportation problems for Initial Basic Feasible Solution 'IBFS'. Like Least Cost Method 'LCM', Northwest Corner Method 'NWCM' and Vogel's Approximation Method 'VAM'. Several methods have been proposed in recent years to solve such difficulties. In this paper, a method named Extended Penalty Method 'EPM' is developed to get a better Initial Basic Feasible Solution 'IBFS' of a transportation problem. In this paper a method have been proposed to checked for balanced transportation problems with no degeneracy. The proposed method Extended Penalty Method 'EPM' obtains a better Initial Basic Feasible Solution 'IBFS' compared to the VAM, which is the optimal solution of the given transportation problem. It consistently provide yields results that are either better or equivalent or a nearly optimal solution.

Key words: Transportation Problem, IBFS, LCM, NWCM, VAM, Optimal Solution.

1 INTRODUCTION:

In today's world everybody wants to minimize the loss and maximize profit to tackle this problem. Different methods of Operation Research proved to be great Mathematical tools. One of its major branches is the transportation problem in linear programming. Which deals with the cost minimization of the problem .For this many methods have been formed and many mathematicians have worked to give rise to new methods. The problem was formalized by the French mathematician Gaspard Monge in 1781. It was first studied by F.L. Hitchcock in 1941, then separately by T.C. Koopmans in 1947 and finally placed in the framework of Linear programming and solved by simplex method by G.B. Dantzig in 1951. Transportation problems are a special kind of LPP. Where the object is to minimize. The cost of distributing a product. from a number of sources to the number of destinations, usually finding the initial basic feasible solution of any transportation problem. There are several methods available to obtain IBFS. North West Method, Least Cost Method, Vogel's Approximation Method. Then finally the optimality of the given transportation problem is checked by the MODI method. Two types of transportation problems balance transportation problems and unbalance transportation problems. There are two phases of transportation problems first find an initial basic feasible solution, the second phase involves optimization of the IBFS which is obtained in phase one. Hlayel A. A. and Alia M. A., Both Hitchcock and Koopman's presentations aided in the transportation strategies that incorporate many shipping sources and multiple destinations. The Transportation problem got its name because many of its applications involved determining how to optimally transport goods [6]. Das et al., discussed the limitation of VAM and improved algorithm by presenting the Logical Development of Vogel's Approximation Method (LD-VAM), and adding a rule if there is the same value of maximal penalty [8]. Sharma N. M. and Bhadane A. P., consider transportation issues, which is an extraordinary sort of direct programming issue. They displayed an elective technique to Northwest Corner Strategy by utilizing a measurable approach, called coefficient of range [9]. Shahi S. and Henry V. V., A new method to find IBFS for transportation problems and compare it with four proposed Mean methods and traditional methods like

NWCM, LCM, and VAM. The approach is simple and uses basic arithmetic and statistics. It is useful for supply chain logistic decision making [14]. According to Bilkour et al. has given an improved method to effectively solve the transportation problem. Modern techniques like a total opportunity cost matrix - zero point minimum method gives better and faster results than older methods like a VAM [17]. Gupta M. and Dayal S., has stated that it is well-known that VAM provides a good IBFS. But it is struggling with the maximum penalty frequency. The aim of this research paper is to Hybridize the VAM method incorporated with the lowest cost entry and northwest corner rule before applying a MODI Method for an optimization [18].

After going through the above-mentioned papers and research developed in the field of transportation problem, the present work have been inicient, which is an improvement on the other well known methods such as VAM.

1.1 TRANSPORTATION PROBLEM FORMULATION

The transportation problem is a type of a linear programming problem, (Kumar 2020) by told that deal with the efficient movement of goods from several supply point to several demand points. The main goal is to minimize the total cost of transportation while meeting all supply and demand requirements. This problem is common in logistics and supply chain management. It helps organizations make better decisions about how to allocate and distribute their resources. Method such as the North West Corner Rule and the Transportation Simplex Method are commonly used to find solutions. Mathematically speaking, the objective of a Transportation Problem is to minimize transportation costs subject to supply and demand constraints. The occurrence of the Transportation Problem arises when there is requirement to transport a singular item from many supply sources (origin) to diverse desires (destinations). Assume that there are n sources of supply ($S_1, S_2, \ldots\ldots\ldots S_n$) and each has X_k ($k=1,2,\ldots\ldots\ldots n$) units of demand. Let C_{kl} represent the cost of transportation one unit of the good along every path from origin k to destination l. The challenge is to identify the Transportation Problem that units shipped per route from origin k to destination l. Mathematically the Transportation Problem may be expressed as a LPP as follows.

$$T = \sum_{k=1}^{m} \sum_{l=1}^{n} C_{kl} Z_{lk} \qquad \text{(objective function)}$$

(Subject to the constraints)

$$\sum_{k=1}^{m} Z_{kl} = X_k, \quad k = 1,2,...,n \qquad \text{(Supply constraints)}$$

$$\sum_{k=1}^{n} Z_{kl} = Y_l, \quad l = 1,2,....,m \qquad \text{(Demand constraints)}$$

$$Z_{kl} \geq 0 \text{ for every } k = 1,2,...,n;\ l = 1,2,...,m \quad \text{(Nonnegative Restrictions)}$$

Where, T = Total transportation cost , X_k = Unit of supply, Y_l = Unit of demand

Z_{kl} = Units shipped per route from source to destination

C_{kl} = Cost for shipping one unit of the commodity from source to destination

This special linear programming formulation of the transportation problem is applicable in various contexts. In such problems, demand and supply are generally represented by integers.

Table 1.1 Transportation Problem's network representation

Sources	Destination				Supply
	1	2	….	n	
1	C11	C12	….	C1n	S1
2 : m	C21 : Cm1	C22 : Cm2	…. …. ….	C2n : Cmn	S2 : Sm
Demand	d1	d2	….	dn	

The table 1.1 illustrates the network representation of transportation problem. Let's suppose that there are n sources and m destinations. The edges connecting sources and destinations serve as a visual representation of the routes among them. The main aspects of the transportation problem are as follows (Diaz-Parra et.al) (2014).

2 MATERIALS AND METHODS

2.1 EXISTING ALGORITHM : FOR VOGEL'S APPROXIMATION METHOD (VAM)

VAM is an improved version of the least cost method that generally has better starting solutions. Following in the algorithm of VAM to find the initial basic feasible solution of transportation problems.

Step[1]: For each row (column), determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the row (column).

Step [2]: Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column)is assigned zero supply (demand).

Step [3]: If exactly one row or column with zero supply or demand remain sun crossed out, stop. If one row (column) with positive supply (demand) remain sun crossed out, determine the basic variables in the row (column) by the least cost method. Stop. If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least cost method. Stop. Otherwise go to step 1.

2.2 PROPOSED ALGORITHM : FOR EXTENDED PENALTY METHOD (EPM)

In this algorithm Extended Penalty Method is developed to find the initial basic feasible solution of a transportation problem. In this method we are only considering balanced transportation problem with no degeneracy. This EPM, provides a very good initial solution. Till now in the existing methods to find the IBFS the best among all is the VAM. Here we are proposing an attempt to make it even more better. The steps of the EPM are given below.

STEP [1]: Select the minimum element of Row (R1). Subtract this element from all the other elements (greater than this element) of R1. This difference will be known as the first set of penalties of R1. Then we will have to select the second lowest value in R1, and then subtract it from the elements; which are greater than the select element value. We will keep on repeating the same process until we will get the last difference of the greatest elements of R1. These differences are called the penalties.

Similarly all the penalties of all the rows (R1,R2,R3,------------Rn) will be found. The number of penalties in each row will be n(n-1)/2
where n = Number of column

STEP [2]: Simultaneously select the minimum element from Column (C1). Subtract this element from all the other elements (greater than this element) of C1. Then we will have to find the second lowest value in C1, and will subtract it from the elements which are greater than the selected element value. Keep on repeating the same process until we get the last difference of the greatest elements of C1.

Similarly all the penalties of all the columns (C1,C2,C3,-------------Cn) will be found. The number of penalties in each column will be m(m-1)/2
Where m = Number of row

STEP [3]: Identity the column or row with the largest Penalty. In case of tie select any one. But it is wise to select the row or column that given the minimum cost allocation.

STEP [4]: Adjust the supply & demand and cross out the satisfied row or column.

STEP [5]: Repeat these steps until all supply and demand value are 0.

3 NUMERICAL EXAMPLE – 1

Consider a Mathematical Model of a Transportation Problem in below.

Table 3.1

	D1	**D2**	**D3**	Supply
S1	6	4	1	50
S2	3	8	7	40
S3	4	4	2	60
Demand	20	95	35	150

Now we find the IBFS of this problem using Extended Penalty Method respectively in below

3.2 SOLUTION OF EXAMPLE 1: USING EXTENDED PENALTY METHOD

Step [1]. Now we find first penalty row and column. The biggest penalty is received on penalty C3, now we will do the allocation on the lowest element of column C3. Adjust the supply & demand and cross out the satisfied row or column.

First Penalty

R1- 5;3;2

R2 -5;4;1

R3 -2;2;0

C1- 3;2;1
C2 -4;4;0
C3 -6*;5;1

Table 3.3

	D1	**D2**	**D3**	Supply
S1	6	4	1[35]	50/15
S2	3	8	7	40
S3	4	4	2	60
Demand	20	95	35/0	150

Step [2]. Now we find second penalty row and column. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2.

Second Penalty

R1- 2

R2-5*

R3- 0

C1- 3;2;1

C2- 4;4;0

Table 3.4

	D1	**D2**	Supply
S1	6	4	15
S2	3[20]	8	40/20
S3	4	4	60
Demand	20/0	95	

Step [3]. Now we find third penalty column C2. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2.

Third penalty

C2- 4;4*;0

Table 3.5

	D2	Supply
S1	4[15]	15/0
S2	8	20
S3	4	60
Demand	95/80	

Step [4]. Now we find fourth penalty column C2. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2. Repeat this steps until all supply and demand value are 0.

Forth Penalty

C2- 4*

Table 3.6

	D2	Supply
S2	8[20]	20/0
S3	4[60]	60/0
Demand	80/20/0	

Total Transportation Cost by EPM is
=3x20+4x15+8x20+4x60+1x35
=60+60+160+240+35
=555
OPTIMALITY CHECK ON EPM
The solution obtained by EPM is optimal.

3.7 SOLUTION OF EXAMPLE 1: USING VOGEL'S APPROXIMATION METHOD

Table 3.8

	D1	**D2**	**D3**	supply	Row penalty
S1	6	4[50]	1	50/0	3 3 - -
S2	3[20]	8[20]	7	40/20/0	4 1 1 -
S3	4	4[25]	2[35]	60/25/0	2 2 2 0
Demand	20/0	95/45/25/0	35/0		
Column penalty	1 - - -	4 4 4 0	1 1 5 -		

Total Transportation Cost by VAM is
=3x20+4x50+8x20+4x25+2x35
=60+200+160+100+70
=590
OPTIMALITY CHECK ON VAM
The solution obtained by VAM is not optimal.

4 NUMERICAL EXAMPLE – 2

Consider a Mathematical Model of a Transportation Problem in below.

Table 4.1

	D1	**D2**	**D3**	Supply
S1	0	2	1	5
S2	2	1	5	6
S3	2	4	3	8
Demand	4	4	11	

Now we find the IBFS of this problem using Extended Penalty Method respectively in below.

4.2 SOLUTION OF EXAMPLE 2 : USING EXTENDED PENALTY METHOD

Step [1]. Now we find first penalty row and column. The biggest penalty is received on penalty C3, now we will do the allocation on the lowest element of column C3. Adjust the supply & demand and cross out the satisfied row or column.

Table 4.3

First Penalty
R1-2;1;1
R2-4;3;1
R3-2,1,1
C1-2;2;0
C2-3;2;1
C3-4*;2;2

	D1	**D2**	**D3**	Supply
S1	0	2	1[5]	5/0
S2	2	1	5	6
S3	2	4	3	8
Demand	4	4	11/6	

Step [2]. Now we find second penalty row and column. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2.

Table 4.4

Second Penalty
R2-4*;3;1
R3-2;1;1
C1-0
C2-3
C3-2

	D1	**D2**	**D3**	Supply
S2	2	1[4]	5	6/2
S3	2	4	3	8
Demand	4	4/0	6	

Step [3]. Now we find third penalty column row and column. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2.

Table 4.5

Third Penalty
R2- 3*
R3-1
C1-0
C3-2

	D1	**D3**	Supply
S2	2 [2]	5	2/0
S3	2	3	8
Demand	4/2	6	

Step [4]. Now we find fourth penalty column C2. The biggest penalty is received on penalty R2, now we will do the allocation the lowest element of Row R2. Repeat this steps until all supply and demand value are 0.

Table 4.6

Fourth Penalty
R3 -1*

	D1	**D3**	Supply
S3	2 [2]	3 [6]	8/2/0
Demand	2/0	6/0	

Total Transportation Cost by EPM is
=1x5+2x2+1x4+2x2+3x6
=5+4+4+4+18

=35

OPTIMALITY CHECK ON EPM

Solution obtained by EPM is optimal.

4.7 SOLUTION OF EXAMPLE 2 : USING VOGEL'S APPROXIMATION METHOD

Table 4.8

	D1	**D2**	**D3**	Supply	Row Penalty
S1	0[4]	2	1[1]	5/1/0	1 1 0 -
S2	2	1[4]	5[2]	6/2/0	1 4 0 0
S3	2	4	3[8]	8/0	1 1 0 0
Demand	4/0	4/0	11/0		
Column Penalty	2 - - -	1 1 - -	2 2 2 2		

Total Transportation Cost by VAM is
=0x4+1x4+1x1+5x2+3x8
=0+4+1+10+24
=39
OPTIMALITY CHECK ON VAM
Solution obtained by VAM is not optimal.

5 RESULTS AND DISCUSSION

After obtaining an **Initial Basic Feasible Solution (IBFS)** by proposed "**Extended Penalty Method**"(EPM) the obtained results are compared with the results obtained by other existing method Vogel's Approximation Method. The results in terms of total transportation cost are given in table 5.1. The results are also checked for optimality (using MODI method) as can be seen in the table 5.1. Table 5.1 comparison of results of Vogel's Approximation Method and Extended Penalty Method (Total Transportation Cost) with optimality check on Extended Penalty Method (EPM).

Table 5.1

S.N.	**VAM**	**EPM**	**MODI**	**Optimality Check on EPM**
1.	**590**	**555**	555	**OPTIMAL**
2.	**39**	**35**	35	**OPTIMAL**

A thorough evaluation of Extended Penalty Method for resolving transportation problem is shown in table 5.1. The total transportation cost obtained through proposed method "Extended Penalty Method" and "Vogel's Approximation Method" is calculated in table to assess the efficacy of the proposed approach. Analysis of table 5.1 data reveals that the proposed approach Extended Penalty Method & MODI consistently yield the same Total Transportation Cost, in almost all the cases. This indicates that Extended Penalty Method performance is comparable in terms of the quality of the answers. The results suggest that the proposed approach Extended Penalty Method may be useful in producing optimum or nearly optimum solutions similar to MODI method. These findings show the Extended Penalty Method possible benefits over the conventional approaches Vogel's Approximation Method in terms of the solution quality

6 Conclusions

In this paper we have developed, a new method named "Extended Penalty Method" (EPM) for finding the IBFS of the transportation problem. The developed method EPM is compared with the well known Vogel's Approximation Method (VAM), in terms of the total transportation cost. When comparing the newly proposed method "Extended Penalty Method" (EPM) to VAM in terms of transportation cost, it consistently yields results that are either better or equivalent to get a direct optimal solution. As can be seen in the table 5.1, the EPM is giving optimal solution in all the examples taken. But this cannot be claimed to given optimal solution always, as EPM is a heuristic method. More research in needed to be done to fulfill this claim.

REFERENCES

[1] Haley K. B., "The solid transportation problem", *Institute for Operations Research and the Management Sciences (INFORMS)*, vol. 10(3), pp. 448, 1962
[2] Ramakrishnan C. S., "An improvement to Goyal's modified VAM for the unbalanced transportation problem", *Journal of the Operational Research Society*, vol. 39(6), pp. 609-610, 1988
[3] Gleyzal A., "An algorithm for solving the transportation problem", *Journal of Research of the National Bureau of Standards*, vol. 54(4), pp. 213-215, 1995
[4] Adlakha V., Kowalski K. and Vemuganti R. R**.**, "Heuristic algorithms for the fixed-charge transportation problem", *Operational Research Society of India*, vol. (43)2, pp. 132-151, 2006
[5] Nikolic I., "Total time minimizing transportation problem", *Yugoslav Journal of Operations Research*, vol. 17(1), pp.125-133, 2007

[6] Deshmukh N. M., "An innovative method for solving transportation problem", *International Journal of Physics and Mathematical Sciences*, vol. 2(3), pp. 86-91, 2012
[7] Hlayel A. A. and Alia M. A., "Solving transportation problem using the best candidate method", *Computer Sciences and Engineering: An International Journal (CSEJI)*, vol. 2(5), pp. 23-30, 2012
[8] Das U. K., Babu A., Khan A. R., Helal A. and Uddin S**.,** "Logical development of Vogel's approximation method (LD-VAM): An approach to find basic feasible solution of transportation problem", *International Journal of Scientific and Technology Research*, vol. 3(2), pp. 42-48, 2014
[9] Sharma N. M., Bhadane A. P., "An alternative method to North West Corner Method for solving transportation problem", *International Journal for Research in Engineering Application & management*, vol. 1(12) pp. 1-3, 2016
[10] Morade N. M., "New method to find initial basic feasible solution of transportation problem using MSDM", *International Journal of Computer Sciences and Engineering*, vol. 5(12) pp. 223-226, 2017
[11] Sengamalaselvi J. , "Solving transportation problem by using MATLAB", *International Journal of Engineering Science and Research Technology (IJESRT)*, vol. 6(1), pp. 371-384, 2017
[12] Arya N. V., Singh P., "An algorithm for solving cost- time trade-off Pair In quadratic fractional transportation problem with impurity restriction", *International Research Journal of Engineering and Technology (IRJET)*, vol. 5(4), pp. 4274-4280, 2018
[13] Mallia B., Das C., Patnaiks S., and Das M., "A new approach to multi-objective transportation problem for binding relationship between warehouse to consumer and consumer to consumer", *International Journal of Intelligent Enterprise*, vol. 8(1), pp.29-43, 2021
[14] Shahi S., Henry V. V., "A work on new method to find initial basic feasible solution of transportation problem in contrast to existing method", *Journal of Algebraic Statistics*, vol. 13(1) pp. 75-81, 2022
[15] Abdelati M. H., Improving solution quality in transportation problem: "A novel algorithm for efficient resource allocation", *AGPE The Royal Gondwana Research Journal of History, Science, Economic, Political and Social Science*, vol. 4(8), pp. 1-10, 2023
[16] Bilkour A. A. S., Henry V. V., Satakshi, "Initial basic feasible solution for transportation problem using TOCM with zero point minimum method", *International Journal of Intelligent Systems and Applications in Engineering*, vol. 12(21s), pp. 3332-3343, 2024
[17] Satakshi and Henry V. V., "A novel approach to find simple, pragmatic solutions to transportation problems", *International Journal of Mathematics in Operational Research, vol.* 27(4), pp. 496-508, 2024
[18] Gupta M. and Dayal S**.,** "Advanced optimization techniques for transportation problem: Hybridization of Vogel's approximation and MODI method", *International Journal of Research and Analytical Reviews (IJRAR), vol. 12(1), pp.126-132,* 2025