Simple experiment to demonstrate the direction of the total acceleration

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***Abstract*— In this article a simple experiment is proposed to demonstrate the direction of the total acceleration of a particle in non-uniform circular motion. With this simple equipment, an experiment was designed to demonstrate the direction of the total acceleration of a sphere as it rolls along a quarter of a circular track. To perform a precise measurement of the variables of interest, the movement of the sphere on the track was recorded and then its displacement was analyzed using the Tracker program, to obtain values of the movement variables at specific points of the trajectory. This proposal is noteworthy due to the scarcity of experimental literature on this topic within mechanical physics.**

***Index Terms*—total acceleration, circular motion, direction, Tracker**

I. INTRODUCTION

In classical mechanics, one of the fundamental topics is kinematics and within this context is the uniformly accelerated circular motion (UACM). With the variables developed in this topic, rotational dynamics can be studied later and therein lies the importance of a concept such as the total acceleration of the UACM. Since this is a vectorial quantity, it has three essential characteristics: magnitude, direction and sense.

In this article we show how with a device of easy construction and with the use of the free downloadable Tracker program, students can perform this practice and check the direction of the total acceleration of a particle rotating in a quarter of a circle.

In the text of our authorship [1] Discussions and experimental activities in general physics page 88, this topic had been addressed, but on that occasion the total acceleration was measured numerically and if it corresponded to the square root of the squares of the tangential and normal acceleration; but due to the movement of a mass in a simple pendulum. In that previous work, the angle's numerical value was not obtained to determine its direction, as is done in the current study with a different apparatus. The results obtained for the measured variables are within the expected margin of error and show that the proposed experiment can be replicated in mechanical physics laboratories.

II. LITERATURE REVIEW

An UACM is circular motion where the particle experiences constant angular acceleration. Fig. 1 shows a particle in circular motion where the accelerations it experiences have been plotted.

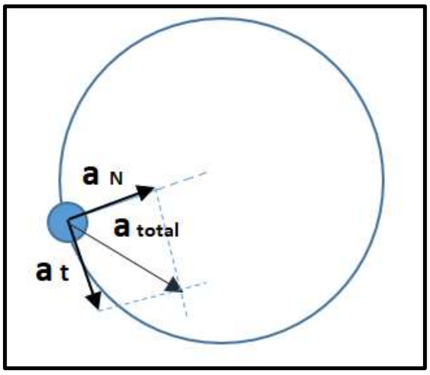

Figure 1. Accelerations in a uniformly accelerated circular motion, Author's own diagram

The angular equations are as follows [2]:

$$\omega = \omega_o + \alpha t \quad (1)$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 \quad (2)$$

$$\omega^2 = {\omega_o}^2 + 2\alpha\theta \quad (3)$$

The accelerations are given by [3]:

$${a_{total}}^2 = {a_N}^2 + {a_t}^2 \quad (4)$$

where

$$a_t = \alpha R \quad (5)$$

and

$$a_N = \frac{v_l^2}{R} \quad (6)$$

with

$$v_l = wR \quad (7)$$

being the linear velocity and w angular velocity.

III. METHODOLOGY DESIGN AND CONSTRUCTION

The methodology of this study consisted of an experimental practice where first the construction and assembly of the equipment to be used was carried out, then an experimental activity was performed and recorded with a camera to later analyze this video in the Tracker program and be able to compare the results obtained with the theoretical calculations. The experimental setup consisted of building a track of a quarter of a circle where a sphere rolls as illustrated in Fig. 2. the track was supported on a wooden base.

Figure 2. Experimental set-up of equipment to be used. Author's photograph

IV. RESULTS AND DISCUSSION

Since the sphere is released from rest at the top of the track, equation (2) can be used to find the angular acceleration α at the quarter circle which gives:

$$\alpha = \frac{(2)(\frac{\pi}{2}\ rad)}{(1.5\ s)^2} = 1.4\ \frac{rad}{s^2}$$

The time was measured with Tracker. Now with the value of the angular acceleration we can obtain the angular velocity of the sphere when leaving the circular trajectory with equation (1) can also be found with the equation (3).

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$$\omega_f = 1.4\ \frac{rad}{s^2} \times\ 1.5\ s = 2.1 \frac{rad}{s}$$

Since the disk has a radius of 0.4 m, the linear velocity can be found as follows equation (7). This value coincides with the value measured with the Tracker program.

$$v_l = 2{,}1\ \frac{rad}{s} \times 0.4\ m = 0.84\ \frac{m}{s}$$

A detailed explanation of the handling of units can be found in our text [1]. To find the normal acceleration we use the equation (6)

$$a_N = \frac{(0.84\ \frac{m}{s})^2}{0.4\ m} = 1.76\ \frac{m}{s^2}$$

And at the point where the sphere leaves the track the tangential acceleration due to the circular motion is given by equation (5):

$$a_t = 1.4\,\frac{rad}{s^2} \times 0.4\,m = 0.56\,\frac{m}{s^2}$$

At the exit point of the track, the total tangential acceleration is the sum of two components acting in the same direction: the tangential acceleration from the circular motion and the acceleration due to gravity. The device was designed so that at that point the earth's gravity of Medellín [4] also acts, therefore:

$$a_t = 0.56\,\frac{m}{s^2} + 9.78\,\frac{m}{s^2} = 10.34\,\frac{m}{s^2}$$

The total acceleration at this point equation (4) is:

$$a_{total} = \sqrt{(1.76\,\frac{m}{s^2})^2 + (10.34\,\frac{m}{s^2})^2} = 10.49\,\frac{m}{s^2}$$

Fig. 3. shows the graph of the acceleration vectors at the point when leaving the track.

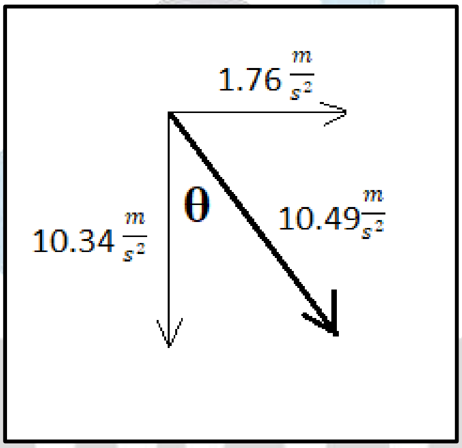

Figure 3. Author's own diagram

The angle θ obtained with the values found is:

$$\theta = tan^{-1}\left(\frac{1.76\,\frac{m}{s^2}}{10.34\,\frac{m}{s^2}}\right) = 9.6°$$

The angle measured with the Tracker program is 6.6° as shown in Fig. 4. this is the interest of this project to demonstrate the direction of the total acceleration. The experiment yielded an angle of 6.6°, which differs from the theoretical value of 9.6° by 31.2%. Although the quantitative result has a high margin of error, the study achieved its qualitative goal of demonstrating the direction of total acceleration. Potential sources of error include friction and the limitations of the video tracking method. Care must be taken with the lower part of the circular track, before performing the experiment, to observe that the sphere falls in a straight line when only gravity acts in order to guarantee that when it travels along the track from the upper part, it does not deviate for any other cause than the accelerations analyzed here.

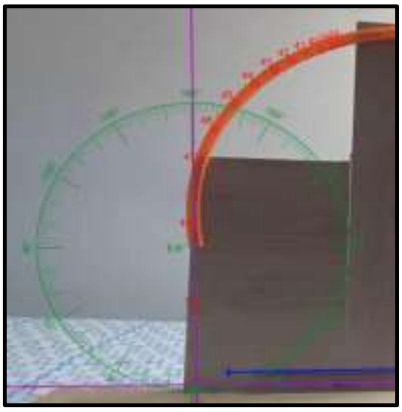

Figure 4. Capture of the online Tracker program where the angle is measured

V. CONCLUSION

With the simple device used, it was possible to demonstrate the direction of the total acceleration at the lower point when the sphere leaves the quadrant of the circular track. At that point, in order to find the total acceleration, some variables were measured with the Tracker program [5] to calculate with them the normal and tangential acceleration acquired by the sphere in the circular motion, but also the acceleration due to gravity is taken into account in the calculation.

It is expected that this proposed device will be useful to other students who are interested in repeating this experience and verify the deflection of the sphere due to the total acceleration acquired at the lower point, this proposal is interesting due to the scarce scientific literature on this subject from the experimental point of view.

VI. ACKNOWLEDGMENT

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