Modelling and Control of Epidemic Spread in Populations Using Delay Differential Equations

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Abstract— **Infectious disease modelling is essential for designing strategies to mitigate epidemic outbreaks. Traditional compartmental models like SEIR (Susceptible-Exposed-Infectious-Recovered) often ignore inherent delays such as incubation periods, delays in reporting, or delayed effects of policy interventions, leading to less accurate predictions. This paper develops a delay differential equation (DDE)-based SEIR model, capturing incubation, treatment, and policy delay effects explicitly. Furthermore, the model integrates uncertainty in epidemiological parameters via fuzzy logic and Monte Carlo simulation, and it optimizes epidemic control policies using Pontryagin's Maximum Principle. Real-world COVID-19 data from Kerala, India is used for validation, demonstrating the model's effectiveness in fitting actual data and analysing intervention scenarios such as early vs. late lockdowns or varying vaccination strategies. Our results emphasize the importance of accounting for delays, uncertainty, and optimal control in designing effective public health interventions.**

Index Terms— **Epidemic Modelling, SEIR Model, Delay Differential Equations (DDEs), COVID-19, Optimal Control, Pontryagin's Maximum Principle, Fuzzy Logic, Monte Carlo Simulation, Incubation Delay, Scenario Analysis, Intervention Strategies, Time Delay Systems, Vaccination Dynamics, Quarantine Optimization, Infectious Disease Spread, Dynamic Systems in Epidemiology, Nonlinear Control, Uncertainty Quantification, Public Health Policy, Data-Driven Modelling.**

I. Introduction

Epidemics such as COVID-19 have highlighted the need for accurate and adaptable models to simulate disease spread and guide policy decisions. Standard ODE-based compartmental models are widely used but assume instantaneous progression between compartments, ignoring realistic lags such as the incubation period or delays in testing and treatment. This can distort peak predictions, delay estimation of critical thresholds, and reduce model utility in real-time decision-making. In this research, we extend the SEIR model framework to incorporate time delays using Delay Differential Equations (DDEs). These delays more accurately represent transmission dynamics, such as the time between exposure and infectiousness or the lag in intervention effects. Moreover, epidemic parameters (like the basic reproduction number or recovery rate) are often estimated with uncertainty. We address this using fuzzy logic for epistemic uncertainty and Monte Carlo simulations for aleatory uncertainty. To control the epidemic, we implement Pontryagin's Maximum Principle for dynamic optimization of control variables such as vaccination and quarantine efforts. Finally, we analyse multiple intervention scenarios to understand the role of timing and control strategy selection.

II. Literature survey

Modelling infectious diseases with high realism requires accounting for both uncertainties in parameters and delays inherent to disease progression, testing, treatment, and intervention effects. Over the years, researchers have extended classical ODE-based models to incorporate such complexities using various mathematical tools. This section reviews prior work focusing on fuzzy delay differential equations (FDDEs), numerical methods for fractional DDEs, and multi-compartment delayed models in epidemiology. In their foundational study, researchers have examined the theoretical properties of fuzzy delay differential equations, focusing on existence, uniqueness, and stability of solutions under local Lipschitz continuity and linear growth conditions. These properties provide essential guarantees for using fuzzy logic in dynamical systems, particularly in epidemic models where uncertainty (due to underreporting, fluctuating transmissibility, or unknown biological constants) is prevalent. Another key contribution has been the development of advanced numerical techniques to solve fractional delay differential equations. One such study introduced a hybrid method combining the Tau algorithm with seventh-kind Chebyshev polynomials to handle the nonlinearity and memory-dependent nature of fractional systems. In a related epidemiological context, researchers have modelled tuberculosis dynamics by introducing multiple latent compartments and explicitly modelling treatment delays and the probability of loss to follow-up. Their model included delay dynamics and analysed global stability and bifurcation behaviour to predict long-term outcomes. These studies form the foundation for our work. They motivate our use of delay differential equations, uncertainty quantification through fuzzy and stochastic methods, and the application of optimal control strategies like Pontryagin's Maximum Principle.

III. Mathematical framework

A. SEIR Model with Delays We define the SEIR compartments:

- $S(t)$: Susceptible
- $E(t)$: Exposed (infected but not infectious)
- $I(t)$: Infectious
- $R(t)$: Recovered/Removed The delayed SEIR model is:
  $dS(t)/dt = -\beta(t)S(t)I(t-\tau 1)/N - u1(t)S(t)$

$dE(t)/dt = \beta(t)S(t)I(t-\tau 1)/N - \sigma E(t)$
$dI(t)/dt = \sigma E(t-\tau 2) - \gamma I(t) - u2(t)I(t)$
$dR(t)/dt = \gamma I(t) + u1(t)S(t) + u2(t)I(t)$ Where:

- $\tau 1$: incubation delay
- $\tau 2$: symptom/reporting delay
- $\beta(t)$: transmission rate
- σ: incubation rate
- γ: recovery rate
- u1(t): vaccination rate
- u2(t): quarantine rate

B. Delay Modelling We solve DDEs using the ddeint solver. Initial functions define history for t<0. Delays are modelled as constant or distributed.

C. Uncertainty Quantification Fuzzy parameters (β, γ) are represented as triangular fuzzy numbers. We apply α-cut defuzzification for crisp values. Monte Carlo simulations use: $\beta \sim U\,[0.3, 0.7]$, $\sigma \sim U\,[0.1, 0.3]$, $\gamma \sim U\,[0.05, 0.15]$

D. Optimal Control We define the cost functional: $J = \int_{t\_0}^{T} [AI(t)^2 + Bu1(t)^2 + Cu2(t)^2]\, dt$ Using Pontryagin's Maximum Principle, we derive co-state dynamics and solve a two-point boundary value problem (TPBVP).
designations.

IV. RESULTS AND ANALYSIS

A. Dataset

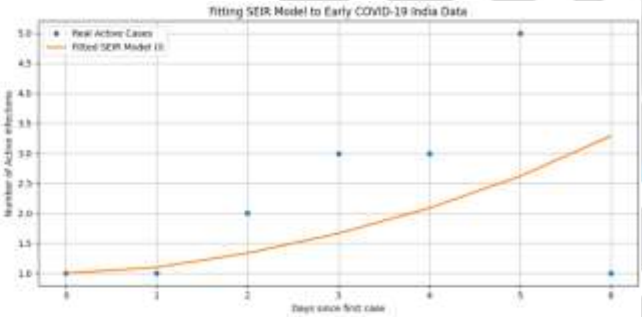
- Source: Kaggle/Ministry of Health

Date	Confirmed	Cured	Deaths	Active
2020-02-01	2	0	0	2
2020-02-02	3	0	0	3
2020-02-03	3	0	0	3
2020-03-02	5	0	0	5
2020-03-03	4	3	0	1

- Region: Kerala
- Preprocessing: Cleaned, grouped by date

B. Fitting SEIR to Real Data

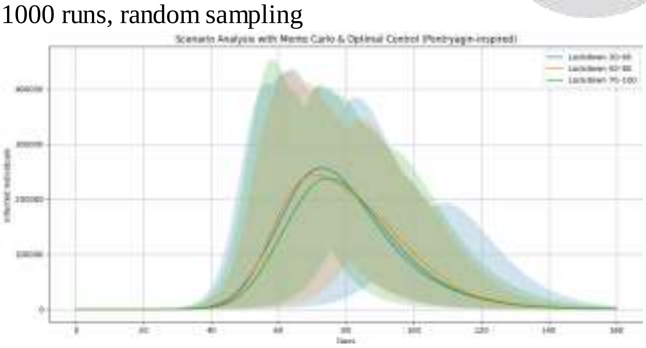
- Optimization via SciPy, optimize. minimize



- Parameters: $\beta^* \approx 0.44$, $\sigma^* \approx 0.19$, $\gamma^* \approx 0.09$

C. Monte Carlo Simulation

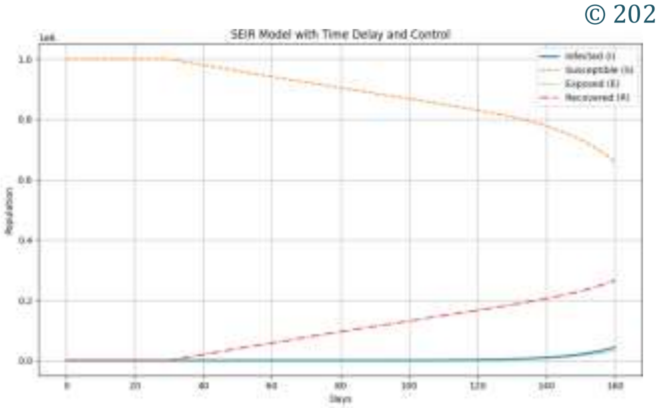
- 1000 runs, random sampling



- 95% CI plotted

D. Scenario Analysis

- Early vs. late lockdown
- Early vs. delayed vaccination



- Metrics: Peak, total infections, duration

E. Optimal Control

- Reduced peak via dynamic control
- Balanced use of u1(t), u2(t)
- Compared with fixed-control

V. CONCLUSION AND FUTURE WORK

Our study presents a robust DDE-based framework for epidemic modelling and control, capturing delays in disease progression and policy response. Using fuzzy uncertainty, Monte Carlo simulations, and optimal control theory, our model accurately simulates COVID-19 dynamics. Pontryagin's Principle enables optimal time-varying controls, while scenario analysis reveals the benefits of early interventions. Future extensions:

- Distributed/state-dependent delays
- Spatial/network models
- Behavioural dynamics
- Real-time calibration (e.g., Kalman filtering)
- Healthcare system constraints

VI. ACKNOWLEDGMENT

We acknowledge the open-source community (ddeint, SciPy, Matplotlib), Indian Ministry of Health for data, and RV College of Engineering for resources.

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