SILOS RESPONSE TO EXPLOSIVE LOADING USING ANSYS

Mesa Manoj Kumar[1] and Dr. Dasarathy A K [2]
[1]P. G Student, Dept. of Civil Engineering, FET, JAIN (Deemed to be University)
[2]Professor, Dept. of Civil Engineering, FET, JAIN (Deemed to be University)

***Abstract* — This study investigates the structural response of cylindrical steel silos subjected to internal explosive loading due to the accidental detonation of stored C-4 material, using finite element analysis (FEA) in ANSYS. Steel silos, commonly employed in various industries for bulk material storage, pose significant safety concerns when used for storing high-energy explosives such as C-4. The research focuses on analyzing the dynamic behavior, stress wave propagation, deformation characteristics, and failure mechanisms of steel silos under internal blast conditions. A detailed 3D finite element model of the steel silo was developed in ANSYS, incorporating material and geometric nonlinearity, appropriate boundary conditions, and internal blast load modeling based on the detonation characteristics of C-4. Multiple charge weights and detonation positions were evaluated to assess the severity and distribution of blast effects. The simulation results highlight the critical zones experiencing high stress, plastic deformation, and potential structural failure, offering insights into the vulnerability and structural resilience of steel silos under such loading. These findings contribute to improved safety design, risk assessment, and structural mitigation strategies for steel storage silos in high-risk applications.**

***Keywords* — Steel silos, C-4 explosive, internal blast loading, finite element analysis (FEA), ANSYS, structural response, dynamic analysis, stress wave propagation, deformation, blast resistance, safety assessment, explosion modeling**

I. INTRODUCTION

A. General

Steel silos, which are widely used to store bulk goods like grains, cement, and chemicals, are essential parts of industrial and agricultural infrastructure. They are a popular option in many industries due to their durability, ease of building, and enormous storage capacity. However, in recent years, researchers and engineers have been investigating how these buildings behave under blast-loading circumstances due to growing worries about their susceptibility to catastrophic occurrences, such as unintentional or deliberate explosions.

Static and seismic loadings have historically received more attention in silo structural design than dynamic phenomena like blast loads. The majority of silos in use today are not designed with the high-intensity, brief overpressures produced by explosions in mind. This results in a serious safety lapse, especially in sectors
where silo systems are used to store combustible or explosive goods.
Applying cutting-edge analysis methods and design concepts that can faithfully model silos' reactions to blast events is crucial to solving this pressing problem. Engineers can forecast structure behavior, locate possible failure zones, and create efficient damage mitigation plans by employing non-linear finite element analysis with programs like ANSYS. Furthermore, using Indian Standards like IS 4991:1968, which offers recommendations for blast-resistant architecture, guarantees that the study is both technically and culturally sound.

By modeling steel silos in ANSYS, applying suitable blast parameters by IS code criteria, identifying critical failure locations, and suggesting design modifications or retrofitting techniques to improve structural resilience, this study seeks to close the gap between traditional silo design and blast-resistance engineering. It is anticipated that the findings of this study will create a framework for evaluating blast resistance in industrial storage structures and help create safer silo designs. ANSYS Explicit Dynamics

A specialized simulation program called ANSYS Explicit Dynamics is used to examine extremely non-linear, transient dynamic phenomena that include material failure, complicated contact interactions, significant deformations, and short-duration loads. It is especially useful for simulating collisions, crashes, explosions, and drop tests. It is a component of the ANSYS Workbench suite. In contrast to implicit solvers, the explicit solver ensures precision and stability in scenarios where conventional approaches might not work by computing the system's reaction over minuscule time increments.

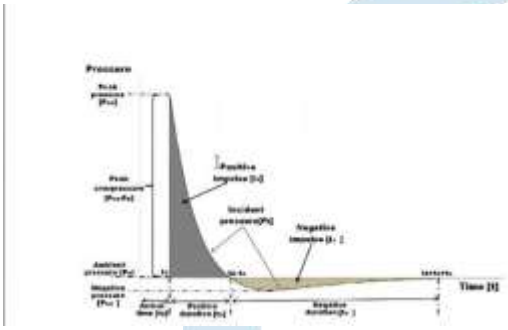


Fig 1 Blast Pressure

B. Introduction to ANSYS & C-4 Explosive Material

ANSYS Explicit Dynamics gives researchers and engineers a strong platform to assess the physical behavior of materials and structures under harsh circumstances by utilizing the power of the finite element method and sophisticated contact algorithms. This capacity is essential for creating systems that are safer and more dependable across sectors including aerospace, automotive, defense, and civil engineering, this expertise is essential for creating safer and more dependable systems.

Introduction to C-4 Explosive Material

Common plastic explosive C-4, Composition C-4, is well-known for its high detonation velocity, stability, and versatility in industrial and military applications. Approximately 91% of its mass is made up of RDX (Research Department Explosive or cyclotrimethylene trinitramine), which is accompanied by binders and plasticizers like polyisobutylene and di(2-ethylhexyl) sebacate, as well as a trace amount of a taggant (for identification and detection purposes).

Because of its malleability, the material can be readily molded to match particular charges or structural elements. Because of its high brisance (breaking capability) and flexibility, it is perfect for controlled demolitions, military breaching, and simulations requiring structural failure due to blast loading. Under typical handling circumstances, there is less chance of unintentional detonation because C-4 is comparatively insensitive to physical shock and friction.

The Jones-Wilkins-Lee (JWL) equation of state, which captures the detonation behavior, pressure-time response, and energy release characteristics of C-4, is a high-explosive material model commonly used in simulation environments like ANSYS Explicit Dynamics. Studying failure causes, pressure distribution, and the dynamic reaction of materials and buildings requires the ability to accurately simulate the propagation of blast waves and their interactions with surrounding structures, which is made possible by the JWL equation.

Designing blast-resistant structures and assessing vulnerability in structural and civil engineering research depends on an understanding of how materials and components behave under explosive loads. Enhancing safety and resilience in engineering design is possible by modeling C-4 in a high-fidelity simulation environment such as ANSYS Explicit Dynamics, which offers vital insights into stress wave propagation, fragmentation, and structural damage patterns.

C. Scope & Benefits

Modeling Steel Silos: Using Ansys software, a comprehensive 3D model of a steel silo was created. Accurate depictions of the silo's geometry, material characteristics, and actual construction details will all be included in this model.

With blast loading as the main focus of analysis, the model will incorporate all pertinent loading circumstances, including static loads (caused by the weight of stored material) and dynamic loads. Application of Blast Loading Parameters (According to IS Codes):

The application of blast loading will be determined by the blast parameters outlined in the pertinent IS standards, such as IS 875 (Part 5) and IS 4991 (for blast-resistant design).
These codes offer precise standards for figuring out blast strength, load duration, and blast distance effects.
To guarantee the correctness of the model, parameters such as impact radius, shock wave propagation, and explosion intensity will be used to replicate real-world situations.

II. LITERATURE REVIEW

"Blast Resistant Design of Structure" S. Sharma, R. Kumar, International Journal of Scientific Research, 2021. This paper reviews various methods applied to different types of structures, such as masonry, concrete, and steel, for blast resistance. It emphasizes the importance of understanding blast loading as dynamic loading and discusses the effectiveness of different design strategies, including the application of IS 4991-1968.

"Blast Analysis and Blast Resistant Design of R.C.C Structures" A. Patel, M. Patel International Journal of Civil Engineering and Technology, 2017, This study presents the effect of blast loads on a 5-story R.C.C. building. Blast loads are calculated manually as per IS 4991-1968, and force time history analysis is performed in STAAD Pro. The research compares the influence of blast loads on the structure to that under static conditions, providing insights into designing structures to resist blast effects.

"Comparative Study of Finite Element Analysis of Steel Silos with Rectangular and I-Stiffeners" P. Singh, R. Verma International Journal of Engineering Research and Technology, 2021 This study compares the structural performance of steel silos with different stiffener configurations under static loads. While it focuses on static analysis, the insights into stiffener effectiveness can inform design strategies for enhancing blast resistance in silos.

"Silos Structural Response to Blast Loading" M. Younes, A. Al-Haddad, et al. Engineering Structures, 2021 This study evaluates the structural response of grain silos to massive blast loads, using the Beirut explosion as a case study. It employs non-linear finite element modeling to simulate the damage observed on-site. The research highlights the lack of existing studies on dynamic loads, especially blast impacts, on silos and provides recommendations for design improvements.

"Experimental Verification of Full-Scale Silo Structure Demolition under Blast Loading" J. Zhang, Y. Liu, et al. Structural Concrete, 2024 This article presents an experimental and numerical study of the collapse behavior of a 60-meter-high cement silo under blast loading. The research provides insights into the failure mechanisms and validates numerical models against experimental data, offering valuable information for designing blast-resistant silos.

"Experimental Study on Blast Resistant Structure with Composite Materials", Advanced Materials Research, 2011 This experimental study investigates the effectiveness of composite materials in enhancing blast resistance. Two sealed steel boxes, one with a rigid and flexible composite layer and the other without protection, were tested under explosive loading. The results demonstrate the potential of composite materials in mitigating blast effects, offering practical solutions for damage reduction in steel structures.

"Blast Resistance of Reinforced Concrete Structures: A Review" (Journal of Structural Engineering, 2020) This review article summarizes the current state of knowledge on blast resistance of reinforced concrete structures. It discusses various aspects, including material properties, structural response, and design methods. The authors highlight the importance of considering non-linear material behavior, geometric non-linearity, and dynamic effects in blast resistance analysis.

"Non-Linear Dynamic Analysis of Blast-Loaded Structures" (International Journal of Protective Structures, 2019) This study presents a non-linear dynamic analysis of blast-loaded structures using finite element methods. The authors investigate the effects of blast loading on structural response, including stress distribution, displacement, and failure mechanisms. They demonstrate the importance of considering non-linear material

behavior and geometric non-linearity in blast resistance analysis. The study provides valuable insights into the development of more accurate blast resistance design methods.

"Blast Resistance Analysis of Steel Frames using Non-Linear Finite Element Methods" (Journal of Constructional Steel Research, 2018) This research investigates the blast resistance of steel frames using non-linear finite element methods. The authors evaluate the effects of blast loading on steel frame response, including member failure, connection damage, and overall structural stability. They demonstrate the effectiveness of non-linear finite element methods in predicting the blast resistance of steel frames. The study provides recommendations for improving the blast resistance design of steel structures.

"Experimental and Numerical Investigation of Blast-Resistant Design for Buildings"(Journal of Explosives and Shock Waves, 2017) This study presents an experimental and numerical investigation of blast-resistant design for buildings. The authors conduct experiments on scaled models and numerical simulations using finite element methods. They evaluate the effectiveness of various design strategies, including reinforced concrete, steel framing, and composite systems. The study provides valuable insights into the development of blast-resistant design guidelines for buildings.

III. OBJECTIVES AND METHODOLOGY

OBJECTIVES

1. To model and perform non-linear analysis of the steel silos in Ansys.
2. To apply the blast parameter per IS codes loading criteria.
3. To identify the failure locations and to provide solutions to reduce the damage level

METHODOLOGY

1. Selection of Silo Geometry and Materials
2. A detailed 3D model of the silo will be developed in Ansys Workbench.
3. Application of Load Cases [Dead Loads & Blast Load Parameters]
4. Failure Identification

IV. STRUCTURAL MODELING AND ANALYSIS

STRUCTURAL MODELING

Diameter of the silos	3500mm
Height of the silos	17520mm
Cone angle	75
Thickness of the silos wall	100mm
The material used to build	Steel structures
Material stored in the silos	milk
Explosive loads add distance	1m, 2m, 3m, 4m

Table1: Geometry of silos

Material	Properties
Steel [Fe500]	
Density	7850 kg/m^3
Youngs modulus	2 x 10^5 Mpa
Poisson ratio	0.3
Bulk modulus[K]	166.67×10^9Pa
Shear modulus[G]	76.92×10^9Pa

Table 2: Material properties

The **Bulk Modulus (K)** of **Fe500 steel** can be calculated using the relation between **Young's Modulus (E)** and **Poisson's Ratio (ν)**:

$$K=E/3(1-2\nu)$$

Where E=200 Gpa, V=0.3

$$K=200/3(1-2(0.3))$$

$$K= 166.67Gpa$$

$$K= 166.67\times10^9Pa$$

The **Shear Modulus (G)** of **Fe500 steel** can be calculated using the relationship between **Young's Modulus (E)** and **Poisson's Ratio (ν)**:

$$G=E/2(1+V)$$

Where, E=200 Gpa, V=0.3

G= 200/2(1+0.3)

G= 76.92 GPa

$G=76.92\times10^{9}$Pa

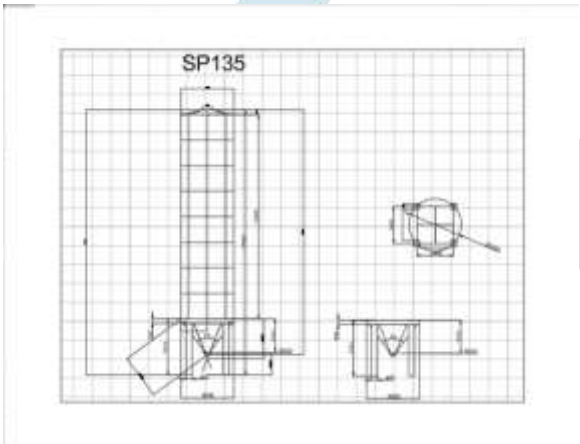


Fig 2 Auto Cad Drawing

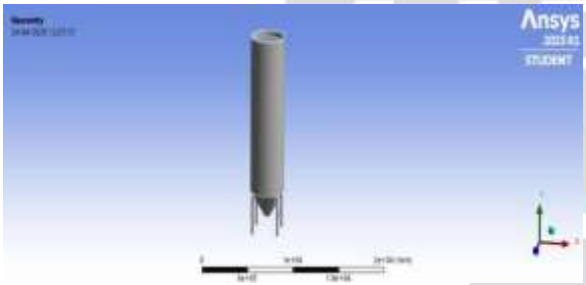


Fig 3 Ansys Model

1.1 Application of Load Cases [Dead Loads & Blast Load Parameters]

- Recommended values of blast pressures for design from IS code [IS 4991 (1968)]

Table 3: C4 Material Properties

Storage material	C4
Density	1601 kg/m^3
C-J pressure	28000000000 Pa
C-J detonation velocity	8193 ms^{-1}

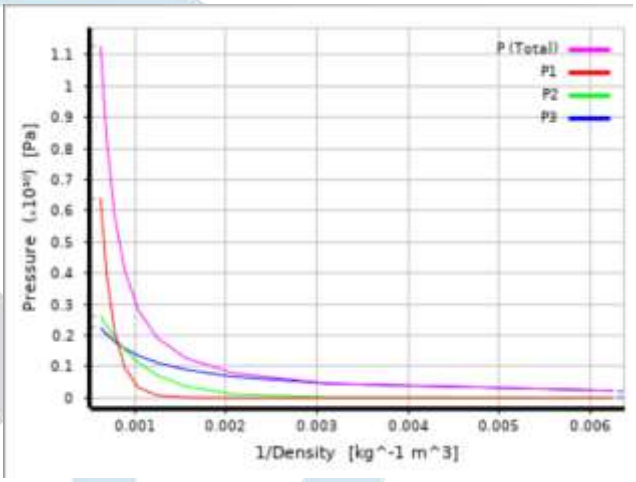


Fig 4 explosive JWL

For **C4 as an explosive**, the **pressure** is usually modeled with the **Jones–Wilkins–Lee (JWL) Equation of State**, used to describe detonation products:

$$P=A\cdot e^{-R1V}+B\cdot e^{-R2V}+(\omega E/V)$$

Where:

- P = pressure (Pa)
- V = relative volume ($V = \rho_0 / \rho$)
- E = internal energy per unit volume (J/m^3)
- A,B,R1,R2,ω = material constants (specific to C4)

C4-specific parameters (typical values, can vary):

- A=6.097×1011 Pa
- B=1.061×1011 Pa
- R1=4.5
- R2=1.1
- ω=0.30
- E=energy per mass × density = 4.9×10^{6}J/kg x 1600kg/m^3

  E= 7.84×10^{9} J/m^3

  $P=6.097\times10^{11}x\ e^{-9}+1.061\times10^{11}\ x\ e^{-2.2}+(0.30\ x\ 7.84\times10^{9}/2)$

$P=75.242958x10^{6}+1.176\times10^{10}+1.176x10^{9}$

P=13,011.24 kPa.

Penetrates through pores under applied pressure. You may use **capillary pressure** models or empirical methods.

Capillary Pressure Formula: $P=2\cdot\gamma\cdot\cos(\theta)/r$

Where:

γ = surface tension (N/m)

θ = contact angle

r = pore radius (m)

γ= (13,011.24 kPa x 1.75m)/2 x 0.2588

γ= 43,510,032Pa·m

γ= 43.51kPa·m

γ =43.51×10^{3}Pa·m

γ =43,510N/m.

ANALYSIS:

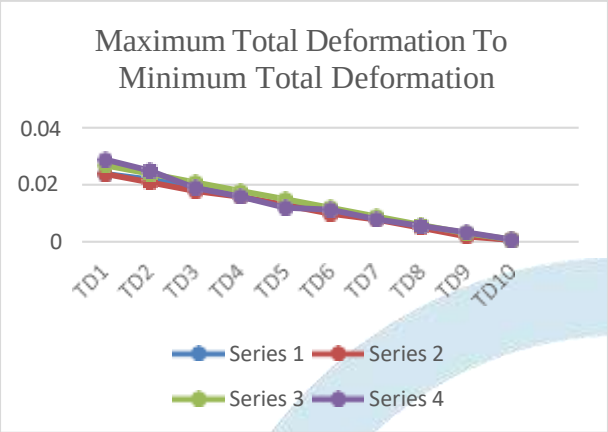
Pressure P	13,011.24 Kpa
Surface Tension	43,510N/m

Table 4: Pressure

Coordinates[x,y,z]

Total Deformation(mm) [1,1,0]	Total Deformation(mm) [2,1,0]	Total Deformation(mm) [3,1,0]	Total Deformation(mm) [4,1,0]
0.024	0.024	0.027	0.029
0.022	0.021	0.024	0.025
0.0193	0.018	0.021	0.019
0.016	0.016	0.018	0.016
0.013	0.013	0.015	0.012
0.0110	0.010	0.012	0.0113
0.0083	0.008	0.009	0.008
0.0055	0.005	0.0060	0.0056
0.0027	0.002	0.00304	0.0034
0.0006	0.0008	0.00085	0.00086

Table 5: Total Deformation of Explicit Dynamic



Graph 1: Maximum Total Deformation to Minimum Total Deformation

Direct Deformation(mm) [1,1,0]	Direct Deformation(mm) [2,1,0]	Direct Deformation(mm) [3,1,0]	Direct Deformation(mm) [4,1,0]
0.019633	0.018	0.023	0.024
0.015218	0.014	0.018	0.018
0.010802	0.0104	0.0125	0.013
0.006386	0.0061	0.0069	0.0070
0.001977	0.0018	0.0013	0.0014
-0.00244	-0.0023	-0.004	-0.0045
-0.00685	-0.006	-0.009	-0.0096
-0.01127	-0.0109	-0.015	-0.0152
-0.01569	-0.015	-0.0210	-0.0211
-0.02010	-0.0194	-0.02665	-0.02765

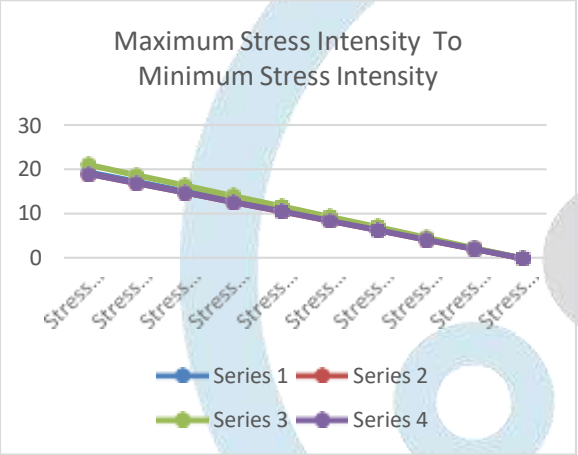
Table 6: Direct Deformation of Explicit Dynamic

Graph 2: Maximum Direct Deformation to Minimum Direct Deformation

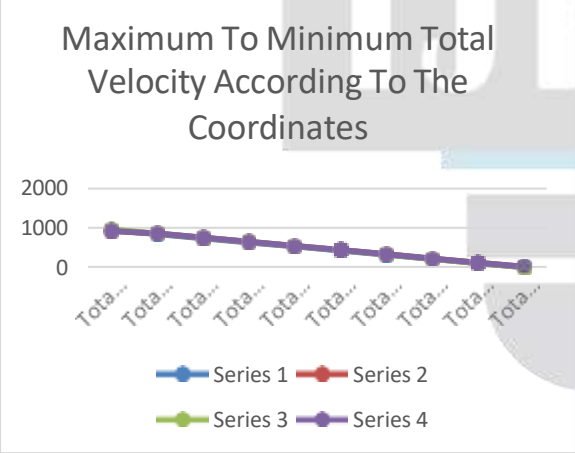
Stress Intensity (Mpa) [1,1,0]	Stress Intensity (Mpa) [2,1,0]	Stress Intensity (Mpa) [3,1,0]	Stress Intensity (Mpa) [4,1,0]
19.4	19.01	21.146	19.01
17.245	16.898	18.797	16.898
15.089	14.786	16.447	14.786
12.933	12.674	14.098	12.674
10.778	10.561	11.748	10.60
8.6223	8.449	9.3984	8.450

6.4667	6.33	7.0488	6.336
4.3112	4.22	4.6992	4.223
2.1556	2.11	2.3496	2.11
0	0	0	0

Table 7: Stress intensity



Graph 3: Maximum Stress Intensity to Minimum Stress Intensity



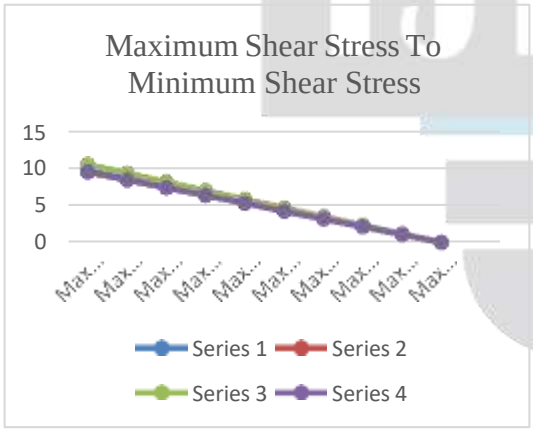
Graph 4: Maximum Total Velocity to Minimum Total Velocity

Maximum Shear stress(MPa) [1,1,0]	Maximum Shear stress(MPa) [2,1,0]	Maximum Shear stress(MPa) [3.1.0]	Maximum Shear stress(MPa) [4.1.0]
9.7	9.5051	10.573	9.50
8.6223	8.449	9.398	8.449
7.5445	7.3929	8.2236	7.399
6.4667	6.3368	7.0488	6.33
5.3889	5.2806	5.875	5.28
4.3112	4.2245	4.699	4.22
3.2334	3.1684	3.52	3.16
2.1556	2.1123	2.34	2.11
1.0778	1.0561	1.17	1.05
0	0	0	0

Table 8: Maximum Shear stress

Total velocity (mm/s) series 1	Total velocity (mm/s) series 2	Total velocity (mm/s) series 3	Total velocity (mm/s) series 4
949.76	923.27	965.88	923.27
844.25	858.6	858.6	858.6
738.75	751.32	751.32	751.32
633.25	644.04	644.04	644.04
527.74	536.76	536.76	536.76
422.24	429.49	429.49	429.49
316.73	322.21	322.21	322.21
211.23	214.93	214.93	214.93
105.73	107.65	107.65	107.65
0.22342	0.37031	0.370	0.37031

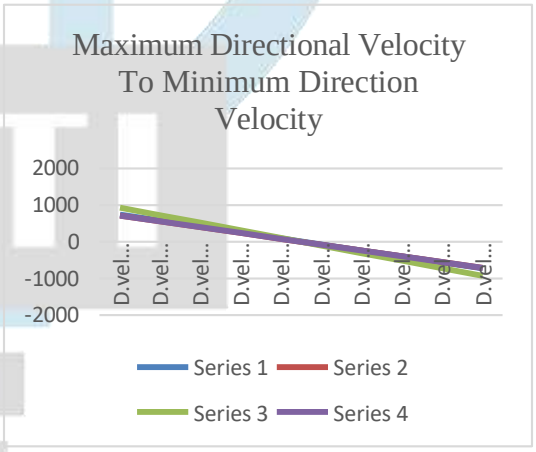
Table 9: Total Velocity



Graph 5: Maximum Shear Stress to Minimum Shear Stress

Directional velocity[mm/s] [1.1.0]	Directional velocity[mm/s] [2,1,0]	Directional velocity[mm/s] [3.1.0]	Directional velocity[mm/s] [4.1.0]
751.11	718.29	938.14	718.29
587.67	559.93	730.5	559.93
424.22	401.58	522.86	401.58
260.78	243.23	315.22	243.23
97.334	84.878	107.58	84.878
-66.111	-73.474	-100.05	-73.474
-229.56	-231.83	-307.7	-231.83
-393	-390.18	-515.34	-390.18
-556.44	-548.53	-722.98	-548.53
-719.89	-706.88	-930.62	-706.88

Table 10: Directional velocity



Graph 6: Maximum Directional Velocity to Minimum Direction Velocity

V. CONCLUSION:

The identification of critical failure locations at the bottom of the silos when the largest deformation values occurred there with an explicit dynamic duration of 5 seconds and an explosion point close to 1 m, 2 m, 3 m, and 4 m with a height of 1 m

The highest possible level of non-linear structural behavior Maximum shear stress is 10.573, maximum directional velocity is 938.14, maximum stress intensity is 21.146, maximum equivalent stress is 19.204, maximum deformation is 0.027, and maximum directional deformation is 0.023.

Suggestions for Damage Reduction: Increase the use of materials with superior ductility and higher yield strength (such as tempered and quenched steels) for improved blast resistance.

To resist localized blast-induced deformation, strengthen the base and cone regions using steel stiffeners, thicker plates, or welded ring reinforcements.

In explosive environments, improved safety standards and design guidelines may lower the chance of structural failures and generally increase worker safety.

REFERENCES:

Aleem Ullah, Furqan Ahmad, Heung-Woon Jang, Sung-Wook Kim, and Jung-Wuk Hong Review of Analytical and Empirical Estimations for Incident Blast Pressure Received May 14, 2015/Revised August 9, 2016/Accepted October 30, 2016/Published Online De cember 12, 2016

Alex M. Remennikov(2003) A Review of Methods for Predicting Bomb Blast E ects on Buildings. JOURNAL OF BATTLEFIELD TECHNOLOGY, VOL 6, NO 3, 2003.

Assal T. Hussein Non-Linear Analysis of SDOF System under Blast Load.European Journal of Scientic Research ISSN 1450-216X Vol.45 No.3 (2010), pp.430-437

Harinath Babu,(2013)Estimation of blast loads, response, and behavior of the structures. International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056 Volume: 05 Issue: 01 Jan-2018 p-ISSN: 2395-0072

H. Dragani, V. Sigmund (2010) Blast loading on structures ISSN 1330-3651 UDC/UDK 624.01.04:662.15 Hrvoje Dragani?, Vladimir Sigmund.

Mohammad M. Abdallah, Bashir H. Osman Numerical Analysis of Steel Building Under Blast Loading, International Journal of Engineering Research and Technology (IJERT) IJERT ISSN: 2278- 0181 IJERTV3IS110846, Vol. 3 Issue 11, November-2014.

M.D. Goel (2001) Blast: Characteristics, Loading and Computation-An Overview CSIR-Advanced Materials and Processes Research Institute (AMPRI), Bhopal 462026, India

Mircea D. Botez, (2015) Lucian Bredean IOP Conference Series: Materials Science and Engineering Numerical Study of an RC Slab Subjected to Blast: A Coupled Eulerian-Lagrangian Approach. IOP Conference Series: Materials Science and Engineering, Vol ume 471, Issue 5

Ngo, T.; Mendis, P.; Gupta, A.; Ramsay, J. Blast Loading and Blast E acts on Structures- An Overview.EJSE Special Issue: Loading on Structures (2007).

Nelson Lam, Priyan Mendis, Tuan Ngo (2004) Response Spectrum Solutions for Blast Loading Advanced Protective Technology for Engineering Structures (APTES) Research Group Department of Civil and Environmental Engineering, The University of Melbourne Parkville 3010, Victoria, Australia.

Remennikov, AM, A Review of Methods for Predicting Bomb Blast E ects on Buildings Journal of Battle eld Technology, 2003, 6(3) 5-10. Published by Argos Press Pty Ltd.

Suryakant S. Birajdar1, Prof. Jawalkar G. C. E ect of Blast Loading on Steel Columns. International Journal of Scientific Engineering and Research (IJSER) ISSN (Online): 2347-3878 Index Copernicus Value (2015): 62.86 Impact Factor (2015): 3.791 UNI

FIEDFACILITIESCRITERIA(UFC); Design of buildings to resist progressive collapse UFC 4-023-03 14 July 2009 Change 3, 1 November 2016.

Syed, Z. I., Mohamed, O. A., Murad, K., and Kewalramani, M. (2017). Performance of Earthquake Resistant RCC Frame Structures under Blast Explosions. Procedia Engineering, 180, 82-90Shalva Marjanishvili, Ph.D., P.E., M.ASCE; and Elizabeth Agnew, M.ASCE Comparison of Various Procedures for Progressive Collapse Analysis. Hinman Consulting Engineers, Inc., One Bush Street, Suite 510, San Francisco, CA 94104.

Vasilis KARLOS George SOLOMOS (2013) Calculation of Blast Loads for Application to Structural Components Administrative Arrangement No JRC 32253-2011 with DG HOME Activity A5- Blast Simulation Technology Development.

Yanchaoshi, Zhong-XianLi, HongHao (2008) A new method for progressive collapse analysis of RC frames under blast loading. Article in Engineering Structures 32(6):1691-1703 June 2010

Bui, L. T., & Nguyen, D. H. (2020). Numerical modeling of reinforced concrete using finite element method in ANSYS. *Case Studies in Construction Materials*, **13**, e00416.[Includes detailed material modeling of concrete, including element types and failure criteria in ANSYS.]

Malek, M. A., Abed, F., & AlHamaydeh, M. (2017). Nonlinear finite element modeling and simulation of plain and FRP-confined concrete cylinders. *Construction and Building Materials*, **147**, 48–61. https://doi.org/10.1016/j.conbuildmat.2017.04.103
Provides mechanical properties and discusses the CDP (Concrete Damage Plasticity) model in ABAQUS, comparable in behavior modeling to ANSYS.

Kadhim, A. K., & Hassan, S. S. (2019). Structural behavior of reinforced concrete deep beams using ANSYS. *IOP Conference Series: Materials Science and Engineering*, **518**, 022056. https://doi.org/10.1088/1757-899X/518/2/022056
Applies ANSYS modeling with various concrete grades and their corresponding material properties.

Mansur, M. A., & Islam, M. S. (2002). Interpretation of stress-strain curves for high- strength concrete. *ACI Materials Journal*, **99**(6), 604–610.
It provides experimental stress-strain data useful for custom material models in FEA.

Lee, J., & Fenves, G. L. (1998). Plastic- damage model for cyclic loading of concrete structures. *Journal of Engineering Mechanics*, **124**(8), 892–900. https://doi.org/10.1061/(ASCE)0733-9399(1998)124:8(892)
Often used as the basis for the CDP model and applicable in ANSYS/ABAQUS.