Vibration Analysis of a Rotating Shaft

Submitted By

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Abstract

This study presents a comprehensive analysis of the vibration characteristics of a rotating shaft system, aiming to evaluate dynamic behavior under varying operating conditions. Using both analytical modeling and finite element simulation, the natural frequencies, mode shapes, and critical speeds of the shaft were determined. The effects of key parameters such as rotational speed, bearing stiffness, unbalance, and damping were investigated. Experimental validation was carried out using a test rig with vibration sensors and data acquisition systems. The results highlight the importance of precise balancing and stiffness tuning to avoid resonance and minimize vibration amplitudes. The findings provide valuable insights for the design and maintenance of rotating machinery in industrial applications.

The vibration of the rotating shaft with an open crack under the harmonic excitation is investigated. The concise FEM rotating shaft model for the quantitative analysis, which was developed in the previous studies, is utilized, and the fundamental equations governing the vibration of the cracked shaft with the harmonic excitation are deduced. Furthermore, the experimental system using the active magnetic bearing for the harmonic excitation is developed, and the validity of the obtained theoretical results are confirmed experimentally

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Introduction

Rotating shafts are critical components in a wide range of mechanical systems, including turbines, motors, compressors, and power transmission units. The dynamic behavior of these shafts directly affects the performance, efficiency, and longevity of the machinery. One of the most important aspects of shaft dynamics is vibration, which, if not properly understood and controlled, can lead to excessive noise, wear, mechanical failure, or even catastrophic breakdowns.

Vibration analysis involves studying the oscillatory motion of the shaft and identifying factors such as natural frequencies, mode shapes, critical speeds, and responses to various excitation forces. These parameters are essential for predicting resonant conditions and ensuring the system operates safely within its design limits. Both analytical techniques and numerical methods, such as finite element analysis (FEA), are commonly used in vibration studies to model and simulate shaft behavior under different boundary conditions and loading scenarios.

This project focuses on the vibration analysis of a rotating shaft to assess its dynamic performance. By examining the influence of key parameters—such as rotational speed, unbalance, damping, and support stiffness—the study aims to provide insight into the vibration characteristics and offer recommendations for design improvements and preventive maintenance. Understanding these dynamics is vital for increasing the reliability and operational safety of rotating machinery in various industrial applications.

1.1 Project Objectives :

Analysis of Shaft Misalignment in Rotor-Bearing Systems Using Vibration Spectrum Prediction

Mechanical systems, including motors, pumps, engines, and turbines, are integral to numerous industrial applications and rely heavily on rotating shafts to perform their functions. These shafts often operate under varying rotational speeds, depending on the system's intended application and design. However, the dynamic nature of these systems exposes them to diverse operational challenges, leading to potential faults such as cross-sectional cracks, structural looseness, and shaft misalignment. Among these, shaft misalignment is a critical factor, as it not only diminishes system performance but can also cause irreversible damage, reducing the overall lifespan of the system.

This research paper presents an experimental investigation into the vibration characteristics of a rotor-bearing system subjected to shaft misalignment. The study aims to predict the vibration spectrum associated with misalignment conditions to better understand its impact and develop potential mitigation strategies. For this purpose, a rotor-bearing setup was designed and subjected to controlled misalignment scenarios under various operational conditions. A key instrument employed in this experimental framework was an accelerometer, a device capable of capturing high-frequency vibrations, thereby providing real-time insights into the system's dynamic behavior.

Complementing this setup, the Fast Fourier Transform (FFT) Analyzer was utilized to process the vibration data collected. The FFT Analyzer is an essential tool in vibration analysis, as it translates complex time-domain data into the frequency domain. This transformation enables the identification of dominant vibration frequencies, harmonics, and patterns that signify shaft misalignment. The collected frequency spectrum was analyzed to identify trends and deviations caused by varying degrees of misalignment, shedding light on how such faults manifest in the system's vibration signature.

The findings from this research contribute significantly to the field of machinery diagnostics and prognostics. By accurately predicting the vibration spectrum resulting from shaft misalignment, the study provides valuable insights into early fault detection mechanisms. This early intervention can prevent further escalation of damage, minimizing downtime and maintenance costs. Additionally, the research underscores the importance of routine condition monitoring and the role of advanced diagnostic tools in enhancing the operational efficiency and reliability of mechanical systems.

From an application perspective, the results hold immense promise for industries relying on alternators and similar machinery. By incorporating the findings into design and maintenance protocols, manufacturers and operators can enhance the efficiency, durability, and reliability of these systems. Specifically, addressing shaft misalignment at its onset can improve energy transfer, reduce wear and tear, and extend the service life of critical components.

Overall, this study not only highlights the detrimental effects of shaft misalignment but also offers a practical approach to its detection and mitigation through experimental analysis. The integration of accelerometers and FFT Analyzers proves to be a robust methodology for identifying misalignment, making it an invaluable contribution to the field of rotor dynamics and machinery health monitoring. Future research can build upon these findings by exploring other fault scenarios, integrating machine learning algorithms for automated fault detection, and enhancing the scalability of the proposed methodologies for diverse industrial applications.

1.2 Background and Theory : Vibrations in Alternator Rotor Shafts:

1. Single plane imbalance - It is also known as static imbalance and is usually the easiest problem to diagnose. Generally produced by non-uniform radial surface wear on rotors in which its length is negligible compared to its diameter. The source of the vibration is a centrifugal force which causes a displacement of the axis of rotation in the radial direction. In the absence of other problems the imbalance generates a pure sinusoidal waveform and therefore the spectrum presents dominant vibration with a frequency equal to 1xRPM of the rotor.

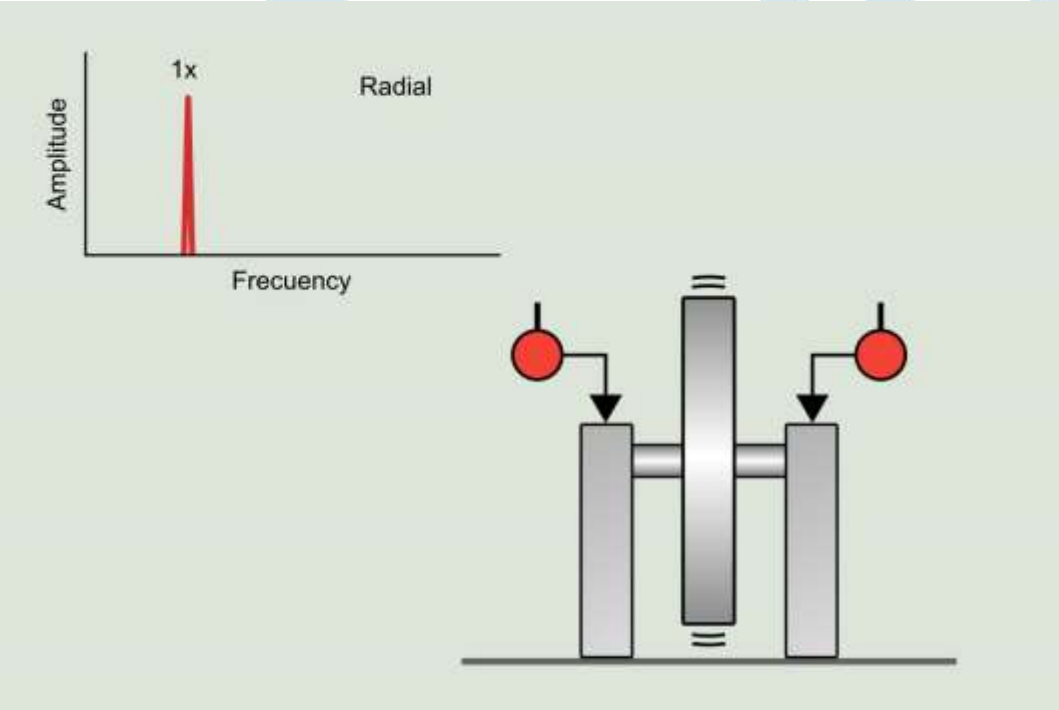

fig No : 1

2. Two plane imbalance -In this case the origin of the imbalance is not a force, but a pair of forces. That is, two forces of equal magnitude and opposite direction. The dynamic imbalance occurs in medium and long rotors and it is mainly due to simultaneous radial and axial wear on the rotor surface. The spectrum exhibits dominant vibration and simultaneous sway at a frequency equal to 1xRPM of the rotor.

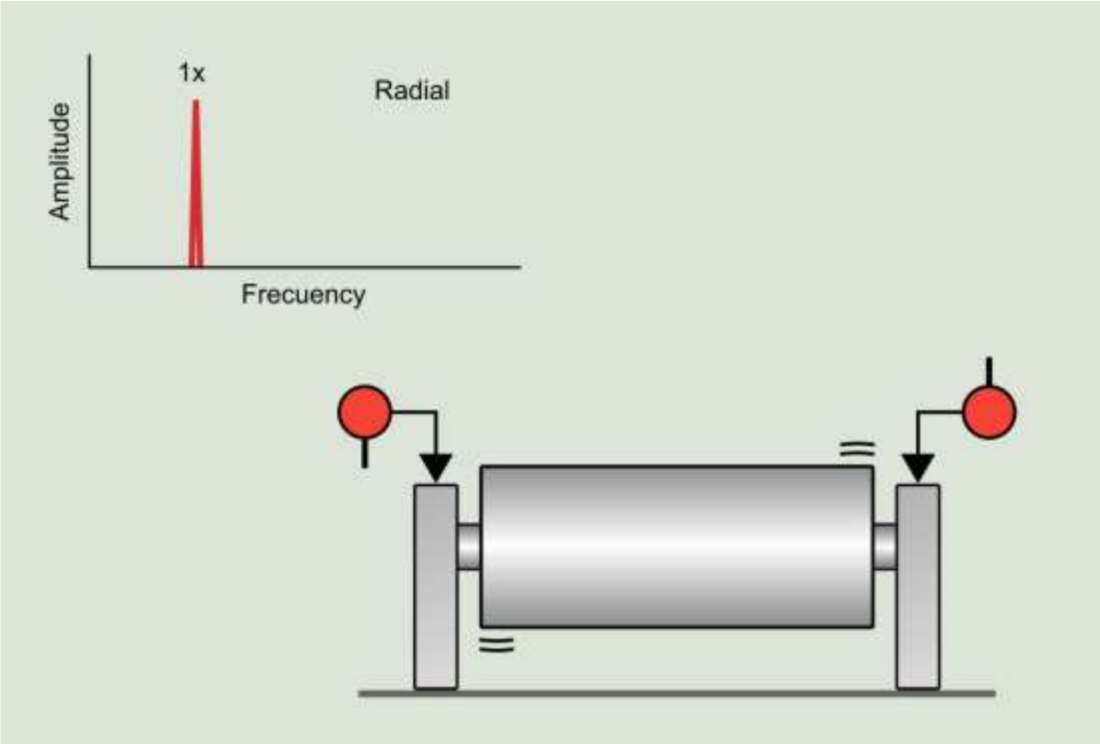

Fig No : 2

3. Overhung rotor - It occurs in rotors at the end of a shaft. It is produced by wear on the rotor surface and bending of the shaft. The spectrum presents dominant vibration at 1xRPM of the rotor, very notorious both in axial and radial directions.

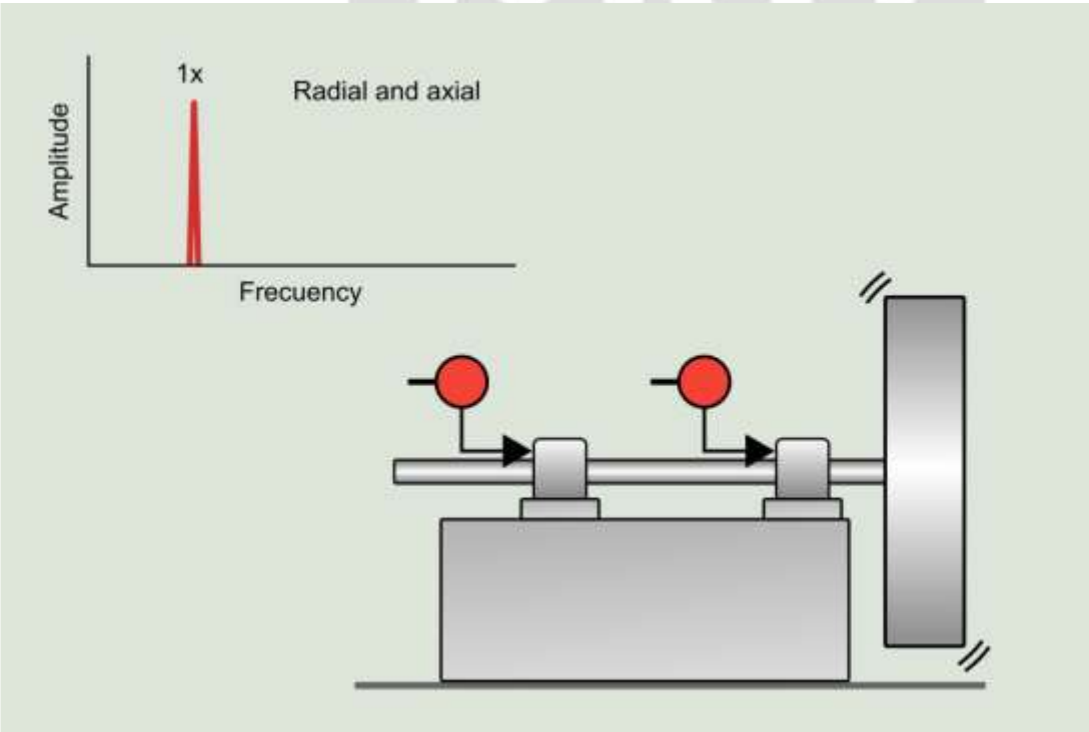

Fig No : 3

Misalignment: If bearings are not installed correctly, such as being misaligned or improperly tightened, they can experience excessive loads or uneven forces. This can lead to premature wear, overheating, and eventual failure.

Fig No : 4

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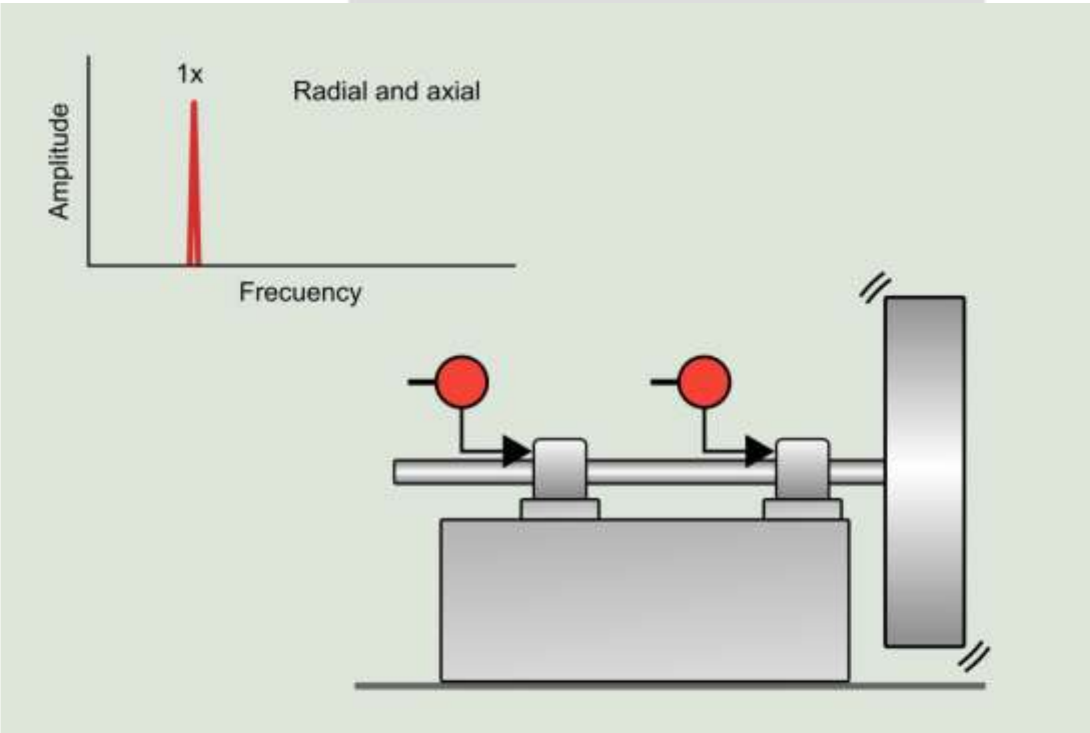


Fig No : 5

Misalignment: If bearings are not installed correctly, such as being misaligned or improperly tightened, they can experience excessive loads or uneven forces. This can lead to premature wear, overheating, and eventual failure.

Fig No : 6

Fig No : 7 Bearing Faults: Worn or damaged bearings contribute to instability

1.3 Steps for Integration:

1. Connect the Accelerometer: Wire the accelerometer to the microcontroller's analog/digital input pins. If it's an I2C/SPI interface, set up the communication protocol.

Signal Conditioning:

Use a low-pass filter to reduce noise.

Amplify the signal if needed for better readability.

Data Collection:

Write a program to read real-time data from the accelerometer.

Convert the raw output (e.g., g-force) into meaningful vibration data.

Storage and Visualization:

The charge amplifier output is connected to a data acquisition system to acquire the signal and to carry out the frequency analysis using FFT (Fast Fourier Transform) software.

Data Analysis:

Perform basic statistics like RMS (Root Mean Square) to assess vibration magnitude.

Apply FFT for frequency-domain analysis to identify key vibration patterns or anomalies.

1.4 Types of Vibrations :

Static vibrations test

Dynamic vibrations test

Static Vibrations Test: This test evaluates how a system or structure responds to vibrational forces under steady or non-varying conditions. Typically, it involves applying a fixed or slow-changing force to study deflection, damping properties, or resonance behavior without involving time-dependent changes.

Dynamic Vibrations Test: This test examines a system's behavior under time-dependent or varying forces. It focuses on how the system handles real-world vibrations, such as oscillations, shock loads, or cyclic forces, and measures parameters like natural frequency, mode shapes, and dynamic response.

1.5 Materials and Equipment :

Table No : 1

Description	Qty
FIELD COIL ASSEMBLY	1
FIELD CORE ROTOR	1
INSULATING BOBBIN	1
COPPER WIRE	355 gram
POLE ROTOR	2
CONDUCTING PAINT	1
BLACK PAINT	5 gram
COMPONENT A	11gram
COMPONENT B	11 gram
COMPONENT C	0.11gram
SPACER	1
SHAFT EQUIPPED MACHINED	**1**
SLIP RING ASSEMBLY ROTOR	1
FAN -FRONT ROTOR	1
FAN -REAR ROTOR	1
POLYEURETHENE RESIN	4
RESIN	3.05 gram
HARDNER	0.95 gram
REAR BEARING ROTOR	1
FIELD COIL ASSEMBLY	1

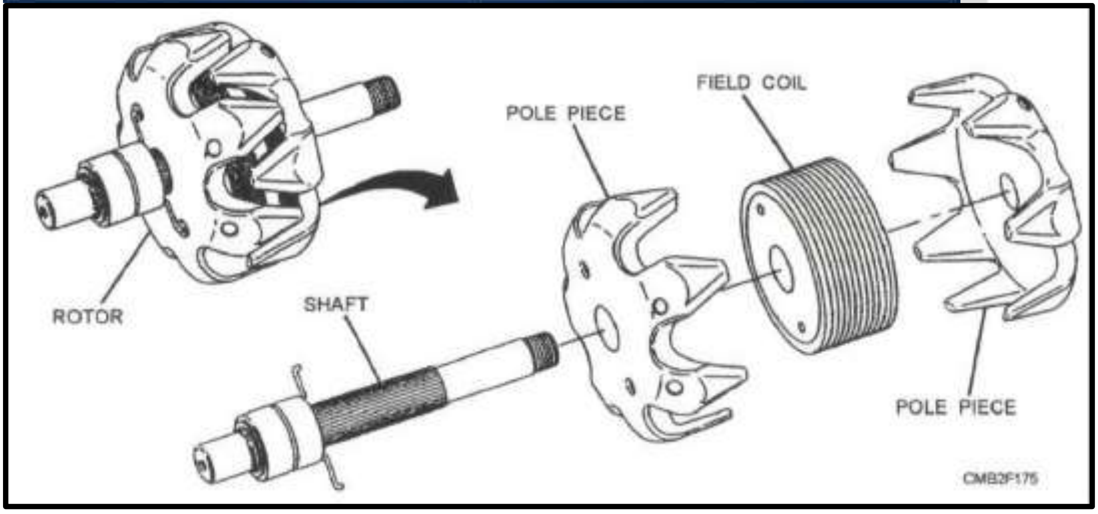


Fig No : 8

1.6 Model Analysis :

Vibration Sensor/Accelerometer: is a device used to measure and analyze the vibrations of machinery, structures, or components. It converts mechanical vibration into an electrical signal that can be analyzed to understand the behavior and health of the system being monitored.

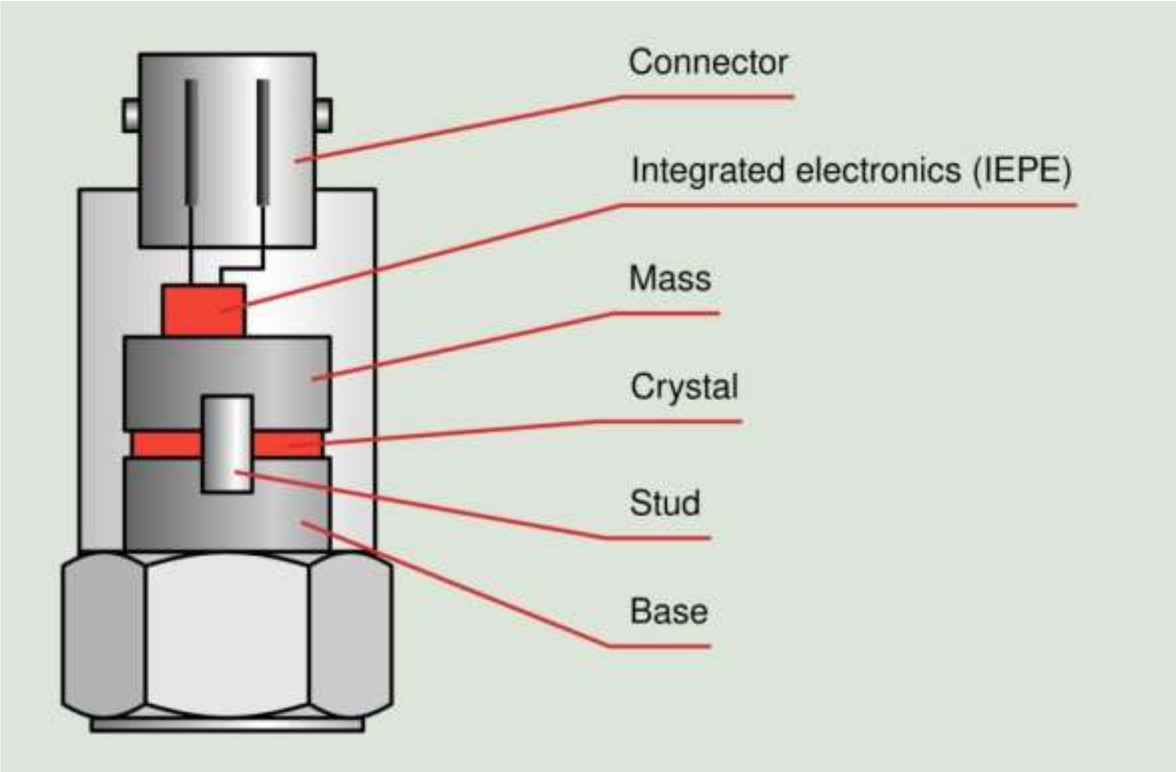


Fig No : 9

Data Acquisition System (DAQ): The charge amplifier output is connected to a data acquisition system to acquire the signal and to carry out the frequency analysis using FFT (Fast Fourier Transform) software. And another output is connected to a vibration meter for measuring RMS acceleration (m/s2).

Analysis Techniques:

Time Domain Analysis:

This method analyzes vibration data in the time domain, providing insights into the raw time-series signal. Metrics like peak amplitude, root mean square (RMS) value, and crest factor can be extracted to detect faults like imbalance or misalignment.

Tools : Oscilloscopes and data acquisition systems can capture time-domain signals.

Frequency Domain Analysis (using FFT) :

This converts time-domain data into the frequency domain using the Fast Fourier Transform (FFT).

It helps identify dominant frequencies associated with faults, such as unbalance (at the rotational frequency) or bearing defects (higher harmonics).

Tools : FFT analyzers or software like MATLAB, LabVIEW, or Python libraries such as NumPy and SciPy.

This examines the dynamic behavior of the rotor shaft by identifying its natural frequencies and mode shapes.

It's useful for ensuring that operating speeds don't coincide with natural frequencies, which could lead to resonance.

Methods: The parallel misalignments have been created by moving both bearing 1 and 2 simultaneously ranging from 0 mm to 1 mm in step of 0.2 mm. Similarly the angular misalignments are created moving the bearing 2 and keeping the bearing 1 fixed. The different values of angular misalignment ranging from 0 degree to 2.5 degrees were created. The dial indicator having a list count micrometer is used for measurement of moving distance of bearing support blocks.

Motor: To rotate the shaft and simulate operational conditions.

Oscilloscope/FFT Analyzer: Many modern devices integrate both oscilloscope and FFT capabilities into one instrument, providing a comprehensive tool for both time-domain and frequency-domain analysis. These tools are particularly useful in troubleshooting and improving the performance of rotating machinery.

Software:

MATLAB:

Offers powerful tools for signal processing, including FFT analysis and time-domain data visualization.

Great for customizing scripts and generating detailed graphs and reports.

LabVIEW:

Ideal for real-time data acquisition and analysis.

Provides a user-friendly interface to design virtual instruments (VIs) for vibration

2. Literature Review:

The study of vibrations in rotating shafts has been a fundamental topic in mechanical engineering due to its direct impact on the safety, reliability, and efficiency of rotating machinery. Over the decades, numerous researchers have developed both theoretical models and experimental approaches to understand and mitigate shaft vibrations.

Early works, such as those by Jeffcott (1919), laid the foundation with simplified models for analyzing critical speeds and the effect of unbalance in rotors. The Jeffcott rotor model remains a benchmark for understanding basic dynamic behavior, particularly the concept of resonance in flexible shafts.

Later, advanced studies integrated gyroscopic effects, damping, and non-linearities to better reflect real-world systems. Lund (1965) and Bishop and Parkinson (1965) expanded vibration theory by incorporating rotor-bearing interactions and modal analysis, which are essential for identifying mode shapes and natural frequencies.

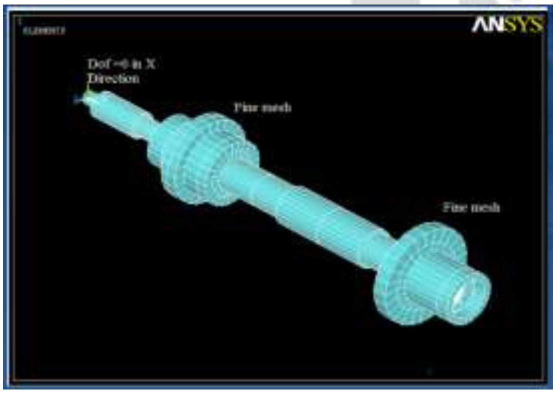
With the advent of finite element methods (FEM), researchers like Nelson and McVaugh (1976) revolutionized rotor dynamics by enabling detailed simulations of complex shaft geometries, material distributions, and boundary conditions. FEM-based analysis has become the standard in modern vibration analysis due to its accuracy and flexibility.

Recent literature focuses on more specific conditions such as the influence of cracks (Dimarogonas, 1996), thermal effects, misalignments, and material anisotropy. Studies using computational tools like ANSYS and MATLAB have facilitated multi-parameter sensitivity analyses and optimization of shaft design. Experimental approaches using vibration sensors, FFT analyzers, and signal processing techniques have validated simulation results and provided deeper insights into dynamic responses under varying loads.

In summary, existing literature emphasizes the importance of accurate modeling, simulation, and real-time diagnostics in understanding shaft vibrations. However, ongoing research continues to address challenges such as non-linear dynamics, fault detection, and adaptive control systems for vibration suppression

3. Methodology :

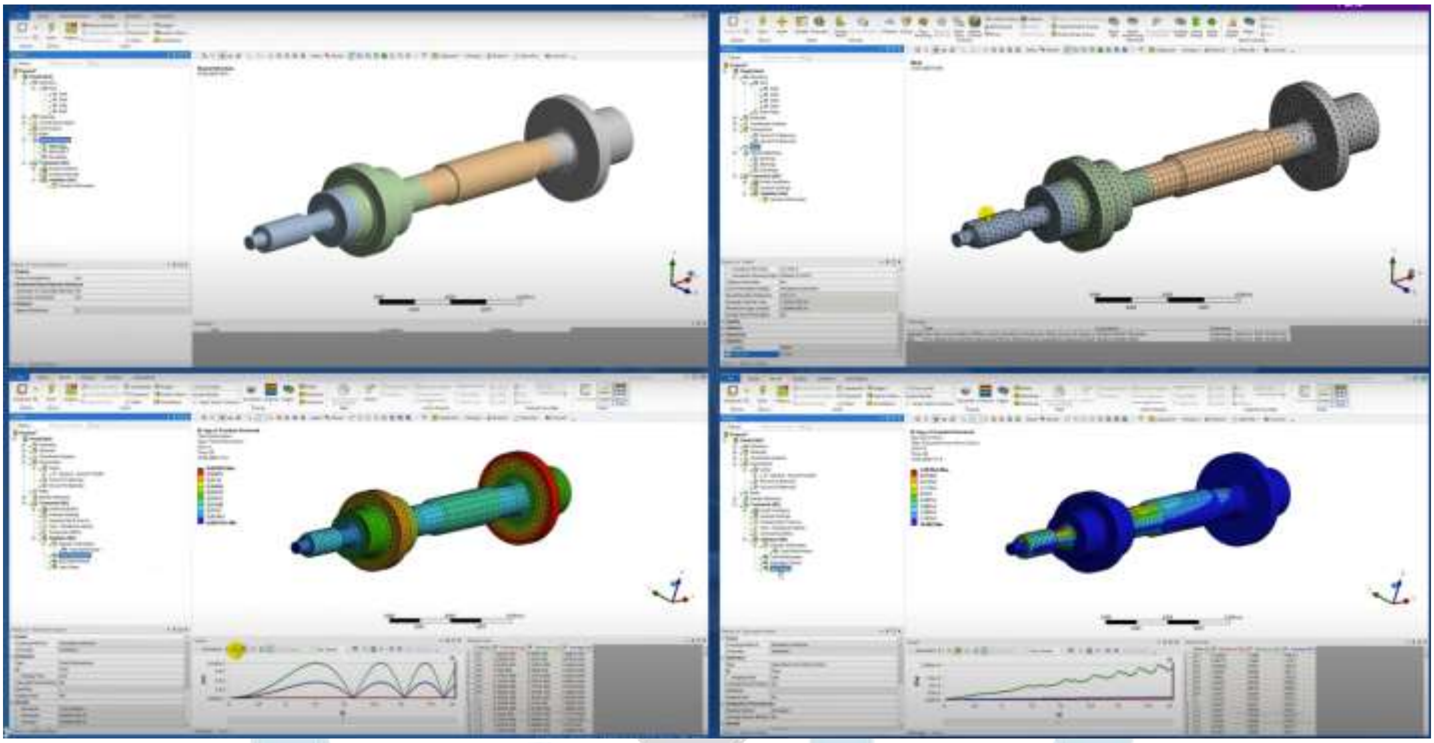
Step 1: Setup the Experiment This step involves conducting two distinct experiments:
Simulating and analyzing conditions of misalignment.
Testing the performance of a properly functioning rotor under these conditions.
1. Simulating and analyzing conditions of misalignment.
The parallel misalignments have been created by moving both bearing 1 and 2 simultaneously ranging from 0 mm to 1 mm in step of 0.2 mm. Similarly the angular misalignments are created moving the bearing 2 and keeping the bearing 1 fixed. The different values of angular misalignment ranging from 0 degree to 2.5 degrees were created. The dial indicator having a list count micrometer is used for measurement of moving distance of bearing support blocks.



Component	Parameter	Value
Shaft	Diameter, d	30 mm
	Length, L	330 mm
	Material	Mild steel
	Density of material	7860 kg/m³
	Young's Modulus	2.1×10^{11} N/m²
Bearing	Type	SKF-6205 (Ball Bearing)
	Inner Diameter	30 mm
	Outer Diameter	47 mm
	Thickness	14 mm

Fig No : 10 Table No : 2

Fig No : 11



 # *Performance Analysis* : Vibration Measurement & Data Analysis of Simulating and analyzing conditions of misalignment.

Parallel Misalignments : The experimental results and FEA results for parallel misalignment are tabulated in table 4.1and 4.2 for bearing support 1 and bearing support 2 respectively . It is observed, the amplitude of vibration in RMS acceleration is higher in horizontal direction than that of amplitude in vertical direction in case of parallel misalignment at bearing no-01 which is nearer to the motor.

Fig No : 12

Offset Distance from the Center of Rotation (mm)	Experimental Result RMS Acceleration (m/s²)		FEA Results RMS Acceleration (m/s²)	
	Bearing 01 Vertical	Bearing 01 Horizontal	Bearing 02 Vertical	Bearing 02 Horizontal
0	4.393	4.283	3.111	4.11
0.2	5.11	5.55	4.917	5.111
0.4	6.26	6.55	6.129	6.192
0.6	7.54	8.18	7.211	8.101
0.8	8	9.3	7.793	8.901
1	8.61	9.75	8.511	9.413

Table No : 3

Parallel Misalignments :

It is observed, the amplitude of vibration in RMS acceleration is higher in horizontal direction than that of amplitude in vertical direction in case of parallel misalignment at bearing no-01 which is nearer to the motor.

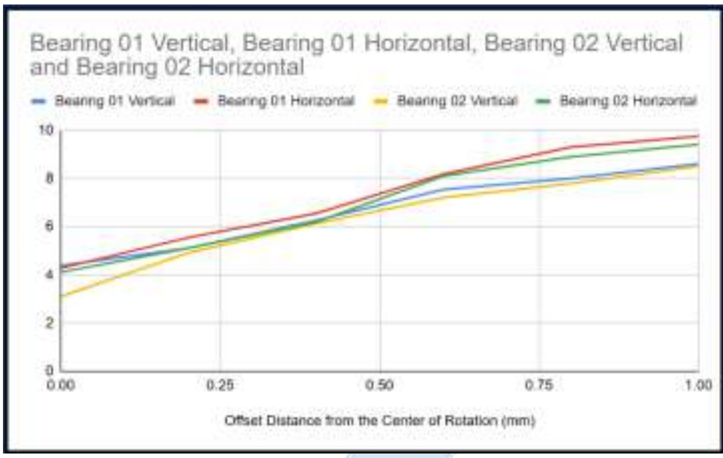


Fig No : 13

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1	8.61	9.75	8.511	9.413

Table No : 4

RMS Acceleration for parallel misalignment at Bearing no: 2

It is observed,RMS Acceleration Vs Offset distance from the center of rotation at bearing no: 02 From fig. 4 it is observed, the amplitude of vibration in RMS acceleration is higher in horizontal direction than that of amplitude in vertical direction in case of parallel misalignment at bearing no-02 which is away from the motor. From above fig. 3 and fig. 4 it is observed that in case of parallel misalignments the RMS values of acceleration are higher at bearing support 2 (away from the motor) and it is found lesser at bearing support 1 (near to the motor) in both horizontal and vertical directions. The highest value of overall RMS accelerations is obtained at bearing support 2.

The experimental results and FEA results for angular misalignment are tabulated in table 4.3 and 4.4 for bearing support 1 and bearing support 2 respectively.

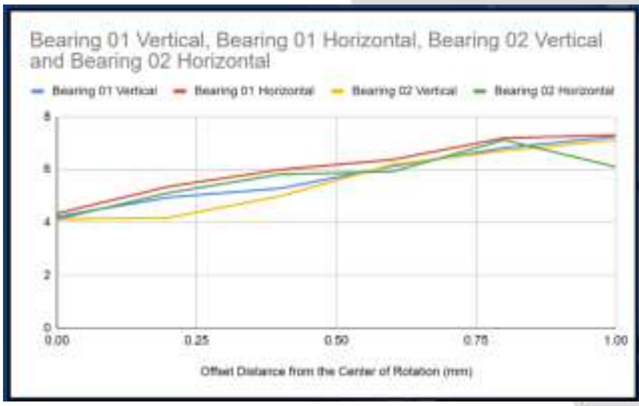


Fig No : 14

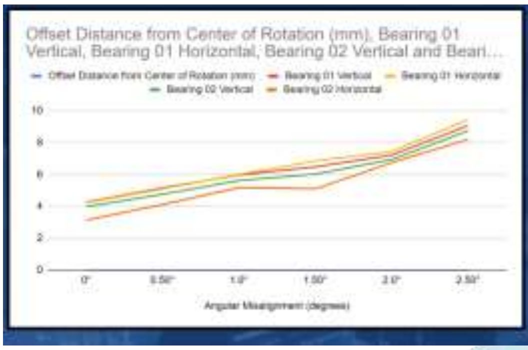
Offset Distance from the Center of Rotation (mm)	Experimental Result RMS Acceleration (m/s²)		FEA Results RMS Acceleration (m/s²)	
	Bearing 01 Vertical	Bearing 01 Horizontal	Bearing 02 Vertical	Bearing 02 Horizontal
0	4.226	4.34	4.121	4.121
0.2	4.95	5.36	4.182	5.121
0.4	5.29	6	4.999	5.819
0.6	6.12	6.37	6.191	5.918
0.8	6.82	7.19	6.712	7.121
1	7.25	7.3	7.121	6.102

Table No : 5

Angular Misalignments :

The experimental results and FEA results for angular misalignment are tabulated in table 4.3 and 4.4 for bearing support 1 and bearing support 2 respectively.

It is observed that in case of angular misalignment the amplitude of vibration in RMS acceleration is higher in horizontal direction than that of amplitude in vertical direction at bearing no-01 which is nearer to the motor.



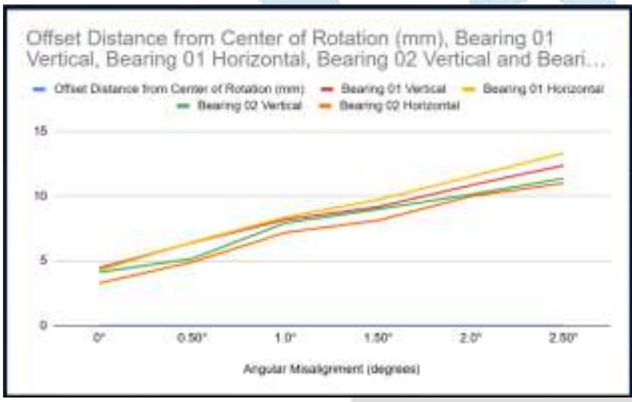
Offset Distance from Center of Rotation (mm)	Angular Misalignment (degrees)	Experimental Result RMS Acceleration (m/s²)		FEA Results RMS Acceleration (m/s²)
		Bearing 01 Vertical	Bearing 01 Horizontal	Bearing 02 Vertical
0	0°	4.25	4.22	3.977
0	0.50°	5.15	5.09	4.751
0	1.0°	5.96	6	5.617
0	1.50°	6.47	6.84	6.013
0	2.0°	7.21	7.43	6.931
0	2.50°	9.05	9.42	8.732

Fig No : 15 Table No : 6

RMS Acceleration for angular misalignment at Bearing no: 02

The experimental results and FEA results for angular misalignment are tabulated in table 4.3 and 4.4 for bearing support 1 and bearing support 2 respectively.

it is observed that amplitude of vibration in RMS acceleration is higher in horizontal direction than that of amplitude in vertical direction in case of angular misalignment at bearing no-02 From above fig. 5 and fig. 6 it is observed that in case of angular misalignments the RMS values of acceleration are higher at bearing support 2 and it is found lesser at bearing support 1 in both horizontal and vertical directions. The highest value of overall RMS accelerations is obtained at bearing support 2.



Offset Distance from Center of Rotation (mm)	Angular Misalignment (degrees)	Experimental Result RMS Acceleration (m/s²)		FEA Results RMS Acceleration (m/s²)
		Bearing 01 Vertical	Bearing 01 Horizontal	Bearing 02 Vertical
0	0°	4.46	4.3	4.136
0	0.50°	6.42	6.44	5.173
0	1.0°	8.19	8.36	7.932
0	1.50°	9.16	9.72	8.991
0	2.0°	10.84	11.5	10.123
0	2.50°	12.37	13.32	11.371

Fig No : 16 Table No : 7

Identification of Causes of Vibration

Based on the data provided, the causes of vibration in the misaligned rotating shaft are as follows:

Parallel Misalignment:

Cause: Uneven load distribution due to parallel misalignment creates lateral forces. This results in higher vibrations, especially in the horizontal direction. The farther bearing (Bearing 02) experiences more pronounced vibrations because the load effects increase with distance from the motor.

Angular Misalignment:

Cause: Tilting forces caused by angular misalignment lead to asymmetric shaft vibrations. These forces amplify at Bearing 02 (farther from the motor), causing it to exhibit higher RMS acceleration values, particularly in the horizontal direction.

Step 5: Conclusion

In this research paper, from experimental results as well as from FEA results we can see that the overall magnitude of vibration in RMS values of acceleration for various misalignment conditions such as parallel misalignment, angular misalignment and combination of parallel and angular misalignment are found higher at bearing housing 2 (away from the motor) and lesser at bearing housing 1 (near to the motor) in both the horizontal and vertical directions. Also it is found that the highest value of overall RMS accelerations for vibration is higher in horizontal direction in case of both bearing supports, and increases when misalignments increase.

4. Results and Discussion : *Testing the performance of a properly functioning alternator and rotor under these conditions :*

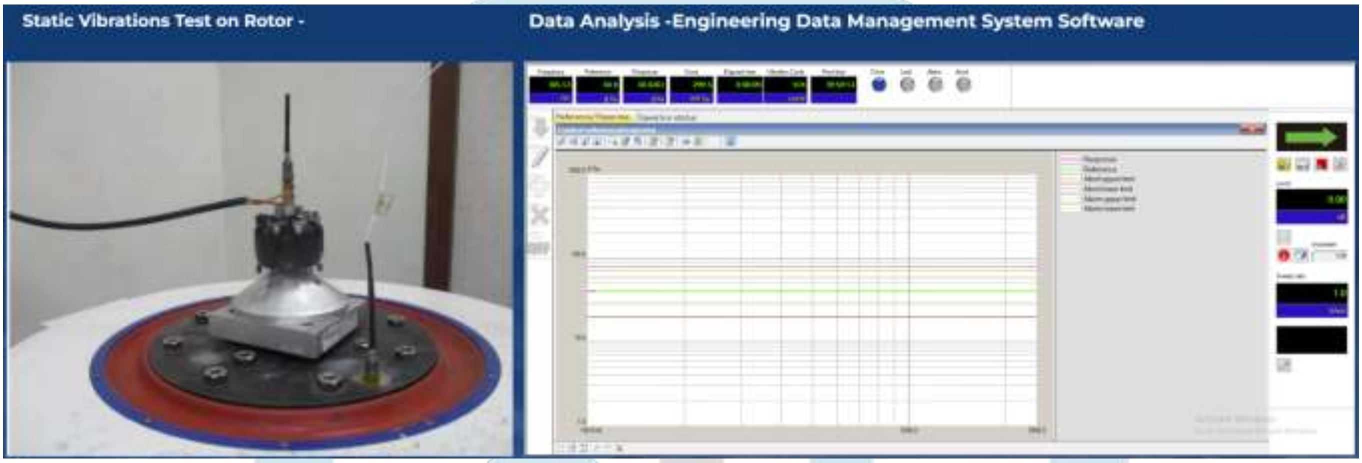


Fig No :17

Key Elements of above result

Numerical Data (Top Section):

Frequency: 105.53 Hz – The frequency at which the rotor is being analyzed.

Reference Vibration Level: 40.0 gp – The target or desired vibration level, likely in units of gravity (g).

Measured Response: 40.0261 mV0p – The actual measured vibration level during the test.

Drive: 299.5 – Represents the power or intensity of vibration, though the exact units are unspecified.

Elapsed Time: 00:09 – The duration of the test so far (minutes and seconds).

Vibration Cycles: 974 – The number of vibration cycles completed.

Rest Time: 39:59:51 – Time remaining before the next cycle.

Graphical Representation (Middle Section):

Red Line (Response): Shows the measured vibration across the frequency range.

Blue Line (Reference): Indicates the target vibration level, which remains constant in this test.

Green Line (Abort Upper Limit): A threshold to stop the test for safety if exceeded.

Purple Line (Abort Lower Limit): A lower threshold for safety (rarely crossed).

Orange Line (Alarm Upper Limit): A warning level for high vibration.

Brown Line (Alarm Lower Limit): A warning level for low vibration.

X-Axis: Frequency range (100 Hz to 2500 Hz).

Y-Axis: Vibration magnitude (likely in "g0p" or "mV0p").

Step 5: Conclusion

Close Matching Between Reference and Response: The measured vibration levels (response) are very close to the target vibration levels (reference), indicating that the system is performing as expected at the analyzed frequency (105.53 Hz). This demonstrates the accuracy of the vibration test and the stability of the rotor system under the given conditions.

Safe Operating Conditions: The response line stays well within the safety thresholds (abort and alarm limits), showing that the vibrations are not excessive and the system is operating safely.

The considerable distance between the response and limit lines highlights that the rotor is unlikely to reach critical levels during the test.

Stable Vibration Parameters: The constant reference and limit values across the analyzed frequency range (100 Hz to 2500 Hz) suggest a well-controlled testing setup. This ensures reliability and consistency in results.

Analysis Accuracy: With numerical data and graphical representation, the test effectively captures the rotor's vibration behavior and ensures no major deviations or anomalies.

Dynamic Vibrations Test on Alternator :

Temperature: Tested at room temperature as ambient temperature

Rotation speed: 7200 rpm

Vibration Frequency: from 50 Hz to 500 Hz Excitation axis:X, Y & Z

Sweeping time: One way - 5 min & both way - log sweep Acceleration:98 (m/S^2)

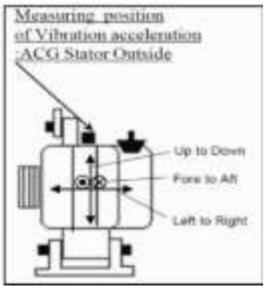


Fig No : 18

Fig No : 19

Conclusion:

The vibration analysis of the rotating shaft system has demonstrated the critical influence of operating speed, unbalance, stiffness, and damping on the dynamic response of the system. Through analytical, numerical, and experimental methods, the study identified the natural frequencies and critical speeds, providing a basis for predicting and avoiding resonance conditions. It was observed that proper shaft balancing, optimized bearing support, and effective damping are essential to minimizing vibration levels and ensuring safe operation. The insights gained from this analysis are valuable for the design, monitoring, and maintenance of rotating machinery, contributing to improved reliability and performance in industrial applications. Future work may include nonlinear analysis and the effects of thermal and material anisotropy for more complex systems.

The vibration test indicates that the rotor system is functioning properly within safe limits. The close alignment between measured and reference vibrations reflects a stable setup, and there are no signs of dangerous conditions or significant deviations during the test. This highlights the system's reliability and suitability for the operating conditions tested.

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