# Matrix Representation of the Sierpinski Pentagon Graph: Analyzing Vertices and Edges at Recursion Depth n

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**Abstract:** In this paper, we're taking a look at the Sierpinski Pentagon Graph to figure out how we can show it using matrices. We're focusing on getting the vertices and edges at different recursion levels 'n'. The Sierpinski Pentagon is a really cool fractal that comes out through repeating a process over and over, making complex geometric patterns. By using matrix theory, we came up with a clear way to map out the connections in the graph. Our research shows how the number of vertices and edges changes as we go deeper into recursion with it getting way more complex. We also touch on how this work might matter more broadly for network theory and computational geometry, hopefully giving some insight into the basic principles behind fractals and basically we're trying to wrap our heads around structured recursive systems better and how to visually show them.

**Keywords:** Sierpinski Pentagon graph, Adjacency Matrix, Incidence matrix, Laplacian Matrix, Recurrence relation.

## Introduction

Sierpinski Pentagon Graph is a fractal structure, which is an extension of a well known Sierpinski triangle and carpet to a pentagonal framework. Fractal graphs are so named because they have self similarity i.e. those graphs retain same structure when magnified further. The Sierpinski graph is a very famous fractal named after Polish mathematician Waclaw's Sierpinski who firstly described the term Sierpinki's triangle in 1915. Construction of Sierpinski pentagon graph: A geometric figure formed by continuing a process and repeating it over and over again. It is a self-similar structure where a process repeats at different levels of iteration. The Sierpinski pentagon graph $S(n, K_5)$ exhibits numerous properties relevant not only to mathematics but also to various domains such as science and economics. This graph emerges through an $n^{th}$ iterative process.

The Sierpinski pentagon graph of $K_5$ with dimension $n$, denoted as $S(n, K_5)$, is characterized by a vertex set $\{1, 2, 3, \ldots, n\}^n$ and edges defined according to specific criteria:

For any vertex pair $(u, v)$, an edge $\{u, v\}$ exists if and only if there exists an index $i \in \{1, 2, 5, \ldots, n\}$ where:

1. $u_j = v_j$ for all $j < i$,
2. $u_i$ ⌸ $v_i$ and $(u_i, v_i) \in E(G)$,
3. $u_j = v_i$ and $v_j = u_i$ for all $j > i$.

Alternatively, if $(u, v)$ represents an edge in $(x, y)$ of $K_5$ and a word "$w$" exists such that $u = wxy\ldots.y$ and $v = wyx\ldots.x$, we consider edge $(u, v)$ to be utilizing edge $(x, y)$ of $K_5$. The graph $S(n, K_5)$ can be constructed recursively from $K_5$ through the following methodology: $S(1, K_5)$ is isomorphic to $K_5$. To derive $S(n, K_5)$ for $n > 1$, replicate $S(n - 1, K_5)$ n times and prepend the letter $x$ to vertex labels in copy $x$ of $S(n - 1, K_5)$. Subsequently, for any edge $(x, y)$ of $K_5$, establish an edge between vertices $xy\ldots.y$ and $vx\ldots x$. For a vertex $x$ of $K_5$, the extreme vertex $x$ of $S(n, K_5)$ refers to the vertex labeled $x\ldots x$.

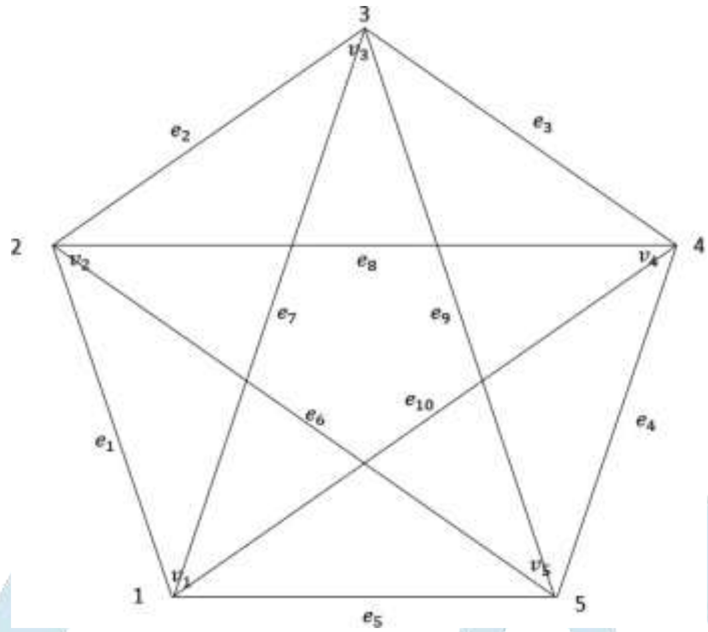


Figure 1: $S(1, K_5)$

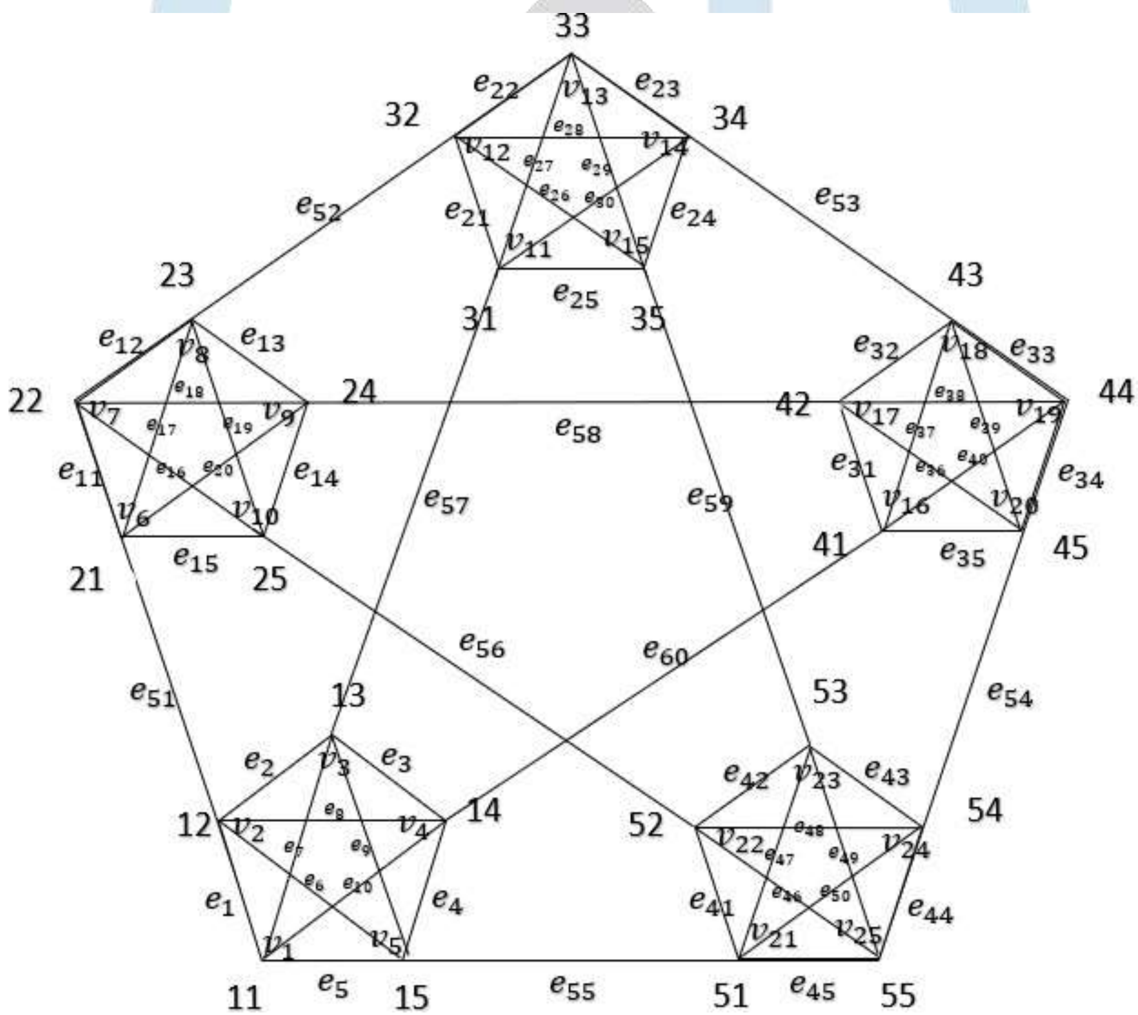


Figure 2: $S(2, K_5)$

The vertices denoted as $\langle i\ldots i\rangle$, where $i \in$ 1, 2, . . . , $n$, are designated as extreme vertices of $S(n, K_5)$. For each $i =$ 1, 2, . . . , $n$, we define $S_i(n+1, K_5)$ as the subgraph of $S(n+1, K_5)$ generated by vertices having the form $\langle i\ldots\rangle$. Vertices that are not extreme in $S(n, K_5)$ are termed inner vertices. Extreme vertices of $S(n, K_5)$ possess degree $K-1$, whereas inner vertices have degree $K$. Evidently, $S_i(n+1, K_5)$ shares an isomorphic relationship with $S(n, K_5)$. As a result, $S(n+1, K_5)$, where $K > 2$, encompasses $n^n$ instances of the graph $S(1, K_5) = n_K$. Those edges within $S(n, K_5)$ that are not contained in any induced $n_K$ are referred to as linking edges.

# Matrix Representation

## Adjacency Matrix Representation

The **adjacency matrix** $A(G)$ for a graph $G$ constitutes an $n \times n$ matrix defined as:

$$A[i][j] = \begin{cases} 1, \text{if vertices i and j are connected by an edge} \\ 0, \text{otherwise} \end{cases}$$

For the **Sierpinski pentagon graph** at the base level $S(1, K_5)$, the adjacency matrix for a complete pentagon (5 vertices) is:

$A(S(1)) =$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| $v_1$ | 0 | 1 | 1 | 1 | 1 |
| $v_2$ | 1 | 0 | 1 | 1 | 1 |
| $v_3$ | 1 | 1 | 0 | 1 | 1 |
| $v_4$ | 1 | 1 | 1 | 0 | 1 |
| $v_5$ | 1 | 1 | 1 | 1 | 0 |

$A(S(2)) =$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ | $v_{16}$ | $v_{17}$ | $v_{18}$ | $v_{19}$ | $v_{20}$ | $v_{21}$ | $v_{22}$ | $v_{23}$ | $v_{24}$ | $v_{25}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $v_1$ | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_2$ | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_3$ | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_4$ | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_5$ | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $v_6$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_7$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_8$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_9$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_{10}$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $v_{11}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_{14}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $v_{15}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $v_{16}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $v_{17}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $v_{18}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $v_{19}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $v_{20}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| $v_{21}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $v_{22}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| $v_{23}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| $v_{24}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| $v_{25}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |

## Incidence Matrix Representation

The **incidence matrix** $I(G)$ of a graph characterizes the relationship between vertices and edges, formulated as:

$$I[i][j] = \begin{cases} 1, & \text{when vertex } i \text{ is incident to edge } j \\ 0, & \text{otherwise} \end{cases}$$

For the **base level S(1, K5)**:

$I(S(1)) =$

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | $e_9$ | $e_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $v_1$ | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| $v_2$ | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| $v_3$ | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| $v_4$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| $v_5$ | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |

**For** $2nd$ **recursion Level** $(2, K_5)$ **matrix, the incidence matrix is-**

$I(S(2)) =$

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $v_{11}$ | $v_{12}$ | $v_{13}$ | $v_{14}$ | $v_{15}$ | $v_{16}$ | $v_{17}$ | $v_{18}$ | $v_{19}$ | $v_{20}$ | $v_{21}$ | $v_{22}$ | $v_{23}$ | $v_{24}$ | $v_{25}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_2$ | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_3$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_4$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_5$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_6$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_7$ | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_8$ | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_9$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{10}$ | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{11}$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{14}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{15}$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{16}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{17}$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{18}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{19}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{20}$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{21}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{22}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{23}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{24}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{25}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{26}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{27}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{28}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{29}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{30}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{31}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{32}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{35}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{34}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $e_{35}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $e_{36}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $e_{37}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{38}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{39}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $e_{40}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{41}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| $e_{42}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| $e_{43}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| $e_{44}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| $e_{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $e_{46}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $e_{47}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| $e_{48}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| $e_{49}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| $e_{50}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| $e_{51}$ | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{52}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{53}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{54}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| $e_{55}$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $e_{56}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $e_{57}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{58}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $e_{59}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $e_{60}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Laplacian Matrix

The **Laplacian matrix** $L(G)$ is expressed by:

$$L(G) = D(G) - A(G)$$

Where:

- $D(G)$ represents the **degree matrix**, a diagonal matrix where each element indicates the degree of the corresponding vertex.
- $A(G)$ denotes the adjacency matrix.

For the **base graph** $S(1, K5)$:

$$D(S(1)) = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$L(S(1)) = D((S(1)) - A(S(1))$$

L(S(1)) = D((S(1)) − A(S(1))

$$L(S(1)) = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

**Similarly for $S(2, K_5)$:**

$$D(S(2)) = \left[\begin{array}{c|ccccccccccccccccccccccccc}
 & v_1 & v_2 & v_5 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{15} & v_{14} & v_{15} & v_{16} & v_{17} & v_{18} & v_{19} & v_{20} & v_{21} & v_{22} & v_{25} & v_{24} & v_{25} \\
v_1 & 4&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_2 & 0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_5 & 0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_4 & 0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_5 & 0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_6 & 0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_7 & 0&0&0&0&0&0&4&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_8 & 0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_9 & 0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_{10} & 0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_{11} & 0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_{12} & 0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_{15} & 0&0&0&0&0&0&0&0&0&0&0&0&4&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_{14} & 0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0&0 \\
v_{15} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0&0 \\
v_{16} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0&0 \\
v_{17} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0&0 \\
v_{18} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0&0&0 \\
v_{19} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&4&0&0&0&0&0&0 \\
v_{20} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0&0 \\
v_{20} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0&0 \\
v_{22} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0&0 \\
v_{25} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0&0 \\
v_{24} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&0 \\
v_{25} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&4
\end{array}\right]$$

$$L(S(2)) = D((S(2)) - A(S(2))$$

$$L(S(2)) = \left[\begin{array}{c|ccccccccccccccccccccccccc}
 & v_1 & v_2 & v_5 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{15} & v_{14} & v_{15} & v_{16} & v_{17} & v_{18} & v_{19} & v_{20} & v_{21} & v_{22} & v_{25} & v_{24} & v_{25} \\
v_1 & 4&-1&-1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_2 & -1&5&-1&-1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_3 & -1&-1&5&-1&-1&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_4 & -1&-1&-1&5&-1&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0 \\
v_5 & -1&-1&-1&-1&5&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0 \\
v_6 & 0&-1&0&0&0&5&-1&-1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_7 & 0&0&0&0&0&-1&4&-1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_8 & 0&0&0&0&0&-1&-1&5&-1&-1&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
v_9 & 0&0&0&0&0&-1&-1&-1&5&-1&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0 \\
v_{10} & 0&0&0&0&0&-1&-1&-1&-1&5&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0 \\
v_{11} & 0&0&-1&0&0&0&0&0&0&0&5&-1&-1&-1&-1&0&0&0&0&0&0&0&0&0&0 \\
v_{12} & 0&0&0&0&0&0&0&-1&0&0&-1&5&-1&-1&-1&0&0&0&0&0&0&0&0&0&0 \\
v_{13} & 0&0&0&0&0&0&0&0&0&0&-1&-1&5&-1&-1&0&0&0&0&0&0&0&0&0&0 \\
v_{14} & 0&0&0&0&0&0&0&0&0&0&-1&-1&-1&5&-1&0&0&-1&0&0&0&0&0&0&0 \\
v_{15} & 0&0&0&0&0&0&0&0&0&0&-1&-1&-1&-1&4&0&0&0&0&0&0&0&-1&0&0 \\
v_{16} & 0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&5&-1&-1&-1&-1&0&0&0&0&0 \\
v_{17} & 0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&-1&5&-1&-1&-1&0&0&0&0&0 \\
v_{18} & 0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&-1&-1&5&-1&-1&0&0&0&0&0 \\
v_{19} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&-1&-1&4&-1&0&0&0&0&0 \\
v_{20} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&-1&-1&-1&5&0&0&0&-1&0 \\
v_{21} & 0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&5&-1&-1&-1&-1 \\
v_{22} & 0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&-1&5&-1&-1&-1 \\
v_{23} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&-1&-1&5&-1&-1 \\
v_{24} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&-1&-1&-1&5&-1 \\
v_{25} & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&-1&-1&-1&4
\end{array}\right]$$

## Result and Discussion

The recursive construction of the Sierpinski pentagon graph is the object of this chapter and the results thus obtained are presented. In particular, we analyze and tabulate the number of vertices and edges mutually at different recursive depth n.

The general recurrence relations for the number of vertices and edges are also formulated and discussed .

$$V(n) = 5^n$$

The above relation indicates an exponential growth of the vertex set as recursion depth increases.
The recurrence relation for the **number of edges** at recursion depth $n$ is:

$$E(n) = \frac{5(5^n - 1)}{2}$$

This relation reflects the rapid and nonlinear expansion of edge connections within the graph as the recursion depth grows.
Using the **matrix-based recurrence relations**, we can efficiently calculate the number of vertices and edges at any recursion depth:

For $n = 1$:

$$V(1) = 5^1 = 5$$

$$E(1) = \frac{5(5^1 - 1)}{2} = 10$$

For $n = 2$:

$$V(2) = 5^2 = 25$$

$$E(2) = \frac{5(5^2 - 1)}{2} = 60$$

For $n = 3$:

$$V(3) = 5^3 = 125$$

$$E(3) = \frac{5(5^3 - 1)}{2} = 310$$

Similarly, we can calculate the number of vertices and edges at any recursion depth 'n'.

## Conclusion

Therefore, we can generalize the Sierpinski graphs $S(n, K_5)$ has $5^n$ vertices and edges $\frac{5(5^n - 1)}{2}$.

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