Mixed Order Reduction Using Honey Badger and Arithmetic Optimization Algorithms for Enhanced System Approximation

Priyajit Dash[1]
Research Scholar
Department of Electronics Engineering
Rajasthan Technical University, Rawatbhata Road, Kota-324010 (Rajasthan), India
dash.priyajit@gmail.com

M. L. Meena[2]
Assistant Professor
Department of Electronics Engineering
Rajasthan Technical University, Rawatbhata Road, Kota-324010 (Rajasthan), India
mlmeena@rtu.ac.in

Abstract: - **In this work, a novel mixed order reduction technique has been proposed using Honey Badger Algorithm (HBA) and Arithmetic Optimization Algorithm (AOA). The denominator of the reduced system is estimated using HBA and the numerator of the reduced system using AOA. Two systems have been tested by the Proposed mixed technique. A comparison of Step Response and frequency response has been presented along with the existing methods. Comparison of Performance Indices and Transient Parameters have been also presented in Tabular form along with the existing techniques. The reduced system using Proposed mixed technique renders excellent close approximation to the HOS than existing techniques.**

Keywords: **Arithmetic Optimization Algorithm, Honey Badger Algorithm, Integral Square Error, Reduced System.**

1. Introduction

Modeling of large-scale dynamical systems is hard to handle because of computation and implementation difficulties, and they are too complex to be used in real-time problems [1]. A system is a large scale when its dimensions are so high, such that conventional techniques of modeling, analysis control, design, and computation fail to give accurate solutions with reasonable computational efforts [2]. The idea behind model order reduction of a system is to change the original system by an approximating system [3]. Hence, for simplification of such systems, the order of these equations must be reduced such that the simplified/lower order system retains all necessary properties of the higher order system. Therefore, for a better understanding of the system, this lower-order system is used to analyze the behavior of the higher-order system by which simulation time reduces and design work becomes easier [4]. The improved clustering algorithm guarantees stability in the reduced model and also preserves the characteristics of the original system in the approximated one [5].

There are several techniques of order reduction available in the literature such as the Atom search optimization technique [6], Bonobo optimizer algorithm [7], Stability equation method [8], bonobo optimizer algorithm, and Stability equation method [9], Renovated pole clustering and Cauer second form technique [10], Eigen permutation and Improved pade approximation [11], differentiation and factor division algorithm [12], Honey badger algorithm and Bonobo optimizer algorithm [13].

2. Problem Description

Consider a HOS of order ‘n’ whose Transfer Function is presented in Eq. (1)

$$G_n(s)=\frac{N_n(s)}{D_n(s)}=\frac{j_0+j_1 s+j_2 s^2+\ldots\ldots+j_{n-1}s^{n-1}}{k_0+k_1 s+k_2 s^2\pm\ldots\ldots+k_n s^n} \quad (1)$$

Where $j_0, j_1, j_2 \ldots\ldots j_{n-1}$ are the coefficients of the numerator of HOS and $k_0, k_1, k_2 \ldots\ldots k_n$ are the coefficients of the denominator of HOS. The main purpose of this work is to approximate the HOS mentioned in Eq. (1) in the form of ROM of order ‘r’($r < n$) in such a manner as to retain all the important features of the HOS whose Transfer Function is represented in Eq. (2).

$$R_r(s)=\frac{N_r(s)}{D_r(s)}=\frac{c_0+c_1 s+c_2 s+\ldots\ldots+c_{r-1}s^{r-1}}{d_0+d_1 s+d_2 s+\ldots\ldots+d_r s^r} \quad (2)$$

Where $c_0, c_1, c_2 \ldots\ldots c_{r-1}$ are the coefficients of the numerator of ROM and $d_0, d_1, d_2 \ldots\ldots d_r$ are the coefficients of the denominator of the ROM.

3. Proposed Mixed Method

The proposed mixed method is divided into two sections, 3.1 and 3.2, where overviews of both optimization techniques are presented.

3.1 Overview of Arithmetic Optimization Algorithm (AOA)

The proposed AOA is a new meta-heuristic method that is inspired by arithmetic operators in mathematics. AOA includes four different operators i.e. Multiplication (M), Division (D), Subtraction (S), and Addition (A). AOA is mathematically modeled to achieve the optimization processes in large search spaces for implementation. Each meta-heuristic algorithm is different from other metaheuristic algorithms. Although differences remain among metaheuristic algorithms in the population-based optimization methods, the optimization process has two main phases one is exploration and the other one is exploitation. The exploration phase covers a wide range of search space by implementing various search agents of an algorithm to avoid local solutions whereas exploitation is the enhancement of the accuracy of the solution in the exploration phase. [14- 15].

3.2 Overview of Honey Badger Algorithm (HBA)

The Proposed HBA imitates the foraging behavior of Honey Badger. This creature loves honey, and it is popular for its fearlessness and intelligence in nature. It likes to remain alone in self-dug holes and meets the other badgers when desires to have a mate. This animal has two different manners of foraging. One manner is to find the prey it looks for by its sense of smell. But it is not enough to smell honey this stage is called the digging stage. Hence getting honey is another way additional birds are called guiding birds it needs when it looks for honey. With the assistance of guiding birds, it gets honey too. This manner of getting the honey with the assistance of a guiding bird is called the honey stage [16-17].

4. Proposed Methodology

The coefficients for both the numerator and denominator of the ROM can be calculated by applying the Proposed optimization technique. The optimal values of the numerator and denominator can be obtained while considering the ISE as the Objective Function. The conventional form of ROM of the second order is mentioned in Eq. (3). However, the simplification of ROM can be found in any order by using the same strategy.

$$R_2(s) = \frac{c_1\, s + c_0}{d_2 s^2 + d_1\, s + d_0} \tag{3}$$

where c_0, c_1 are the unknown coefficient of the numerator and d_0, d_1 and d_2 are unknown coefficients of the denominator, respectively. ISE has been used as an objective function by minimizing its value for finding the ROM mentioned in Eq. (4).

$$ISE = \int_0^{t_{sim}} [X(t) - X_r(t)]^2 dt \tag{4}$$

Where X(t) and $X_r(t)$ represent the step responses of the HOS that have been considered and the ROM obtained using the Proposed mixed method respectively. t_{sim} represents simulation time. The error which is the difference between X(t) and $X_r(t)$ used to estimate the Performance Indices as well as determine the effectiveness of the Proposed methodology using the mixed method.

5. Results and Discussion

In this section two HOS have been used to test the feasibility of the Proposed mixed method where two recent Optimization techniques (AOA & HBA) have been used for order reduction order considering objective function ISE.

.

System 1 Consider a HOS of 4th order [10] presented in Eq. (5)

$$G_4(s) = \frac{28s^3 + 496s^2 + 1800s + 2400}{2s^4 + 36s^3 + 204s^2 + 360s + 240} \tag{5}$$

The denominator of the reduced system has been computed using HBA and presented in Eq. (6).

$$N(s) = 12.2891s + 11.9 \tag{6}$$

The numerator of the reduced system has been computed using AOA and presented in Eq. (7).

$$D(s) = s^2 + 2.0985s + 1.1922 \tag{7}$$

So the reduced 2nd order system presented in Eq. (8)

$$R_2(s) = \frac{12.2891s + 11.9}{s^2 + 2.0985s + 1.1922} \tag{8}$$

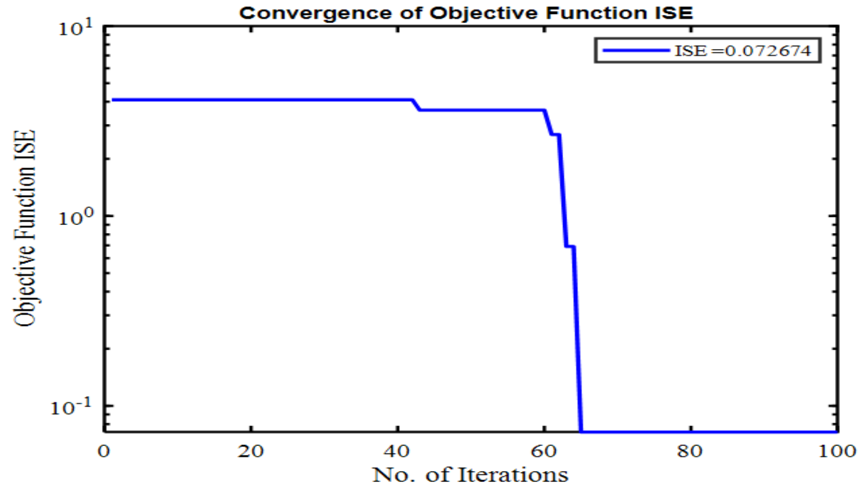

Fig. 1 Iteration graph for System 1

In system 1 HOS of the 4th order system has been reduced 2nd order. The Proposed reduced system of the second order has been compared with the previously published techniques from the literature [8, 10, 11, 12]. The results showed that the reduced system produced by the Proposed mixed method provided a better close approximation when compared to the other four existing techniques from the literature. The iteration graph is depicted in Fig. 1, The comparison of Step response and Frequency response of both the HOS and the Proposed reduced system have been, shown in Fig. 2 and Fig. 3. Comparison of Performance indices have been presented in Table 1. It can be noted that all Performance indices values calculated using the Proposed mixed method were found to be lower than those obtained from the existing techniques. In addition to this, a comparison of the Transient parameters between the HOS and the Proposed Reduced system with the existing techniques has been presented in Table 2. The Proposed Technique outperforms the previously established Techniques in the literature [8, 10, 11, 12].

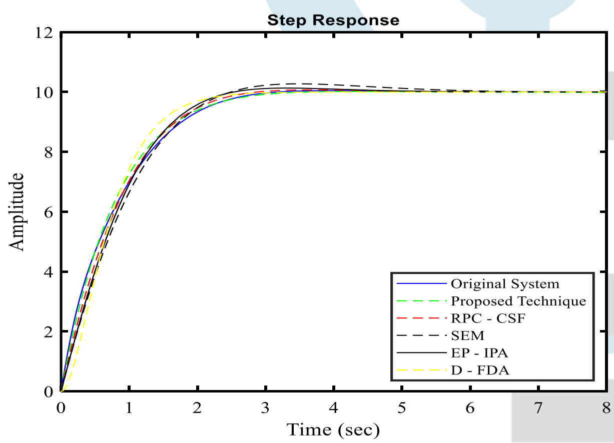

Fig. 2 Step response comparison for System 1

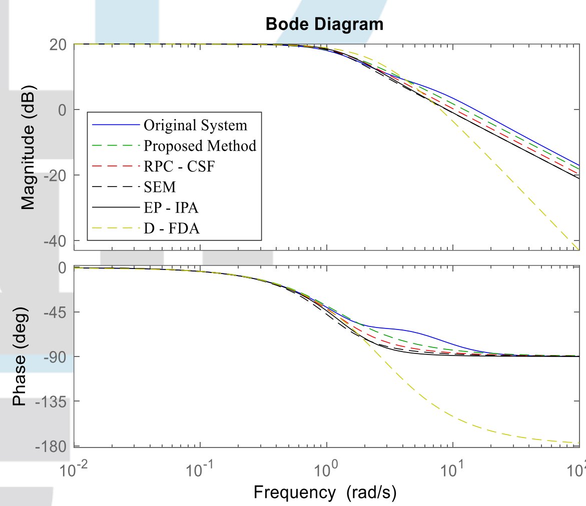

Fig. 3 Frequency response comparison for System 1

Table 1 Comparison of Performance Indices for System 1

Methods	Reduced System	ISE	ISTE	IAE	ITAE
Proposed Method	$R_2(s) = \frac{12.2891s + 11.9}{s^2 + 2.0985s + 1.1922}$	0.0729	0.0813	0.4681	0.8904
RPC – CSF [10]	$R_2(s) = \frac{10.252s + 17.064}{s^2 + 2.305s + 1.7064}$	0.1464	0.0966	0.5680	0.7562
SEM [8]	$R_2(s) = \frac{1800s + 2400}{204s^2 + 360s + 240}$	0.5794	0.6685	1.4254	2.9528
EP – IPA [11]	$R_2(s) = \frac{8.808s + 19.0083}{s^2 + 2.3066s + 1.9007}$	0.3715	0.3092	0.9753	1.4469
D – FDA [12]	$R_2(s) = \frac{70.588}{s^2 + 5.2941s + 7.0588}$	1.1035	0.7601	1.5077	1.8014

Table 2 Comparison of Transient Parameters for System 1

Methods	Rise Time (Sec .)	Settling Time (Sec.)	Max. Overshoot (%)	Peak Value	Peak Time (Sec .)
Original system	1.6920	2.5514	0.4434	10.0443	3.9422
Proposed Method	1.5932	2.5625	0.3425	10.0157	4.3451
RPC – CSF [10]	1.5765	2.3826	0.6890	10.0689	3.6761
SEM [8]	1.6149	4.3379	2.7116	10.2712	3.4969
EP – IPA [11]	1.5282	2.2148	1.2871	10.1294	3.3142
D – FDA [12]	1.2572	2.1801	0	9.9993	4.5059

System 2 Consider a HOS of 8th order [4] presented in Eq. (9)

$$G_8(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \quad (9)$$

The denominator of the reduced system has been computed using HBA and presented in Eq. (10).

$$N(s) = 17.0383s + 5.2926 \quad (10)$$

The numerator of the reduced system has been computed using AOA and presented in Eq. (10).

$$D(s) = s^2 + 6.8702s + 5.2599 \quad (11)$$

So the reduced 2nd order system presented in Eq. (12)

$$R_2(s) = \frac{17.0383s + 5.2926}{s^2 + 6.8702s + 5.2599} \quad (12)$$

In system 2 HOS of the 8th order system has been reduced 2nd order. The Proposed reduced system of the second order has been compared with the previously published techniques from the literature [4, 8, 10, 11]. The iteration graph is depicted in Fig. 4, The comparison of Step response and Frequency response of both the HOS and the Proposed reduced system with existing techniques have been, shown in Fig. 5 and Fig. 6. Comparison of Performance indices have been presented in Table 3. It can be noted that all Performance indices values calculated using the Proposed mixed method were found to be lower than those obtained from the existing techniques. The comparison of Transient parameters of the Original HOS and the proposed reduced system with the existing techniques has been presented in Table 4. The Proposed mixed Technique outperforms the previously established Techniques in the literature [4, 8, 10, 11].

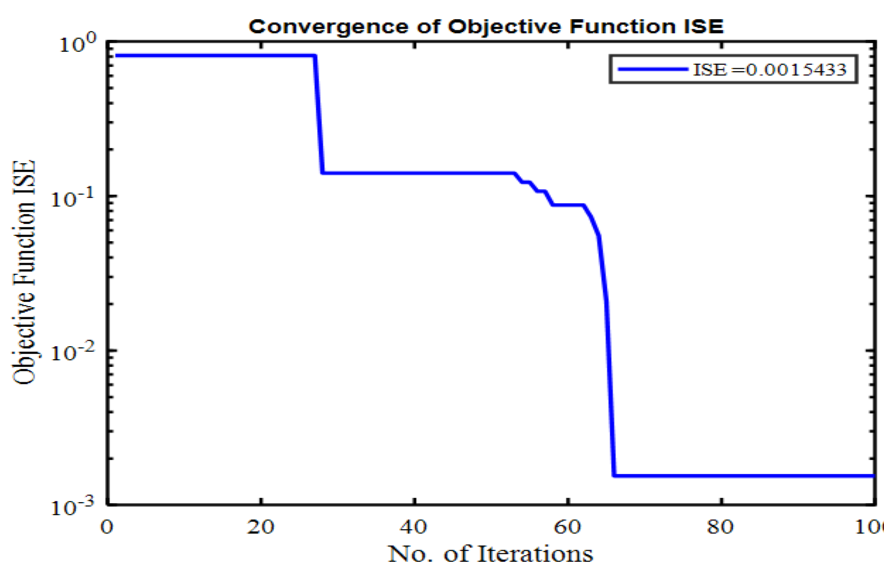

Fig. 4 Iteration Graph for System 2

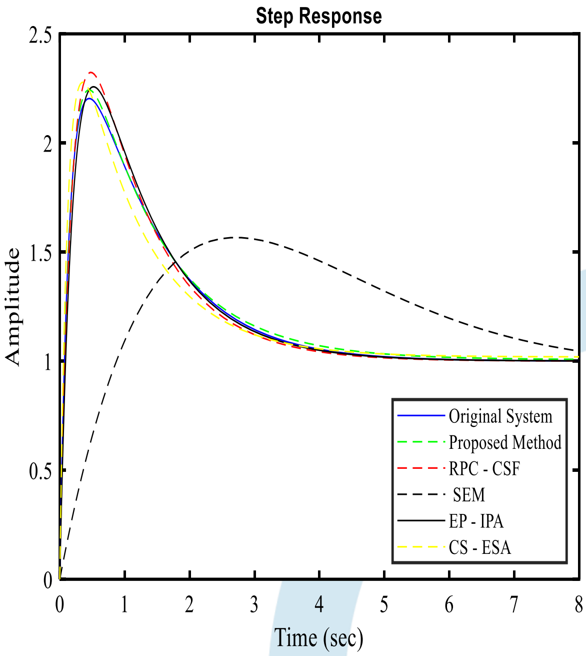

Fig. 5 Step response comparison for System 2

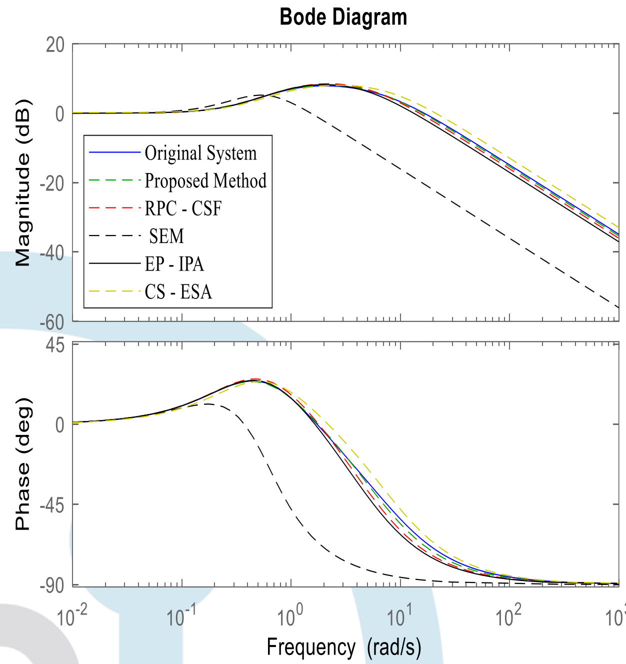

Fig. 6 Frequency response comparison for System 2

Table 3 Comparison of Performance Indices for System 2

Methods	Reduced System	ISE	ISTE	IAE	ITAE
Proposed Method	$R_2(s)=\frac{17.0383s+5.2926}{s^2+6.8702s+5.2599}$	0.0015	0.0043	0.0955	0.3445
RPC – CSF [10]	$R_2(s)=\frac{15.7551s+5.1613}{s^2+6.0039s+5.1613}$	0.0097	0.0087	0.1615	0.2701
SEM [8]	$R_2(s)=\frac{185760s+40320}{118124s^2+109584s+40320}$	2.3942	2.9436	3.1160	7.6449
EP – IPA [11]	$R_2(s)=\frac{14.0097s+4.5}{s^2+5.5s+4.5}$	0.0101	0.0048	0.1282	0.1375
CS – ESA [4]	$R_2(s)=\frac{22.51s+8.151}{s^2+9s+8}$	0.0322	0.0345	0.3337	0.7180

Table 4 Comparison of Transient Parameters for System 2

Methods	Rise Time (Sec .)	Settling Time (Sec.)	Max. Overshoot (%)	Peak Value	Peak Time (Sec .)
Original System	0.0569	4.8201	120.3496	2.2035	0.4490
Proposed Method	0.0595	5.0879	122.9296	2.2432	0.4457
RPC – CSF [10]	0.0628	4.4662	132.3190	2.3232	0.4824
SEM [8]	0.6863	8.7173	56.6128	1.5661	2.6806
EP – IPA [11]	0.0710	4.6828	125.7127	2.2571	0.5117
CS – ESA [4]	0.0457	4.4003	123.5585	2.2778	0.3569

6. Conclusion

This article presents a novel order reduction technique in which the numerator is computed using AOA and denominator coefficients are determined using HBA. The supremacy of order reduction using the proposed technique has been tested by two HOS. The comparison of step response and frequency response of the proposed reduced system has been made with the existing techniques and it depicts that the proposed mixed method gives excellent close approximation to the HOS than the existing techniques. The comparison of performance indices has been presented in Tabular form and the value of ISE is quite lower than existing techniques. Additionally, a comparison of Transient Parameters of HOS and reduced systems with other existing techniques has been presented. The proposed reduced system gives more promising results than existing techniques.

REFERENCE

[1] S. K. Tiwari and G. Kaur, "An improved method using factor division algorithm for reducing the order of linear dynamical system," *Sadhana - Acad. Proc. Eng. Sci.*, vol. 41, no. 6, pp. 589–595, 2016, doi: 10.1007/s12046-016-0499-2.

[2] D. K. Sambariya and O. Sharma, "Model Order Reduction Using Routh Approximation and Cuckoo Search Algorithm," Journal of Automation and Control vol. 4, no. 1, pp. 1–9, 2016, doi: 10.12691/automation-4-1-1.

[3] S.Lavania and D. Nagaria, "Genetic algorithm and Padé-moment matching for model order reduction," *Jnternational J. Electr. Comput. Energ. Electron. Commun. Eng.*, vol. 9, no. 6, pp. 605–609, 2015.

[4] A. Sikander and R. Prasad, "New technique for system simplification using Cuckoo search and ESA," *Sadhana - Acad. Proc. Eng. Sci.*, vol. 42, no. 9, pp. 1453–1458, 2017, doi: 10.1007/s12046-017-0710-0.

[5] R. Komarasamy, N. Albhonso, and G. Gurusamy, "Order reduction of linear systems with an improved pole clustering," Journal of Vibration and Control 18(12) 1876–1885, 2011, doi: 10.1177/1077546311426592.

[6] R. Kumar and A. Sikander, "A new order abatement method based on Atom search optimization," *Int. J. Dyn. Control*, 2022, doi: 10.1007/s40435-022-01094-1.

[7] P. Dash, G. Parmar, and A. Sikander, "A novel strategy for order abatement based on parameter estimation. *Microsyst Technol,* 29, 553–562, 2022,. https://doi.org/10.1007/s00542-022-05406-5

[8] D. K. Sambariya and G. Arvind, "High Order Diminution of LTI System Using Stability Equation Method," Journal of Advances in Mathematics and Computer Scienc vol. 13, no. 5, pp. 1–15, 2016, doi: 10.9734/BJMCS/2016/23243.

[9] P. Dash, M. L. Meena, G. Parmar, and A. Sikander, "A new mixed order reduction method using bonobo optimizer and stability equation. *Microsyst Technol* , 31, 787–800, 2025. https://doi.org/10.1007/s00542-024-05767-z.

[10] A. Kumari and C. B. Vishwakarma, "Order Abatement of Linear Dynamic Systems Using Renovated Pole Clustering and Cauer Second Form Techniques," *Circuits, Syst. Signal Process.*, vol. 40, no. 9, pp. 4212–4229, 2021, doi: 10.1007/s00034-021-01673-1.

[11] J. Singh, K. Chatterjee, and C. Vishwakarma, "System Reduction by Eigen Permutation Algorithm and Improved Pade Approximations," *Waset.Org*, vol. 8, no. 1, pp. 124–128, 2014, [Online]. Available: http://www.waset.org/publications/9997666.

[12] C. B. Vishwakarma and R. Prasad, "Order reduction using the advantages of differentiation method and factor division algorithm," *Indian J. Eng. Mater. Sci.*, vol. 15, no. 6, pp. 447–451, 2008.

[13] P. Dash, M. L. Meena, and A. Sikander, "Order Abatement of Linear System using Mixed Method," *Int. Conf. Adv. Smart, Secur. Intell. Comput.*, pp. 1–9, *2024,* doi: 10.1109/ASSIC60049.2024.10508001.

[14] L. Abualigah, A. Diabat, S. Mirjalili, M. Abd Elaziz, and A. H. Gandomi, "The Arithmetic Optimization Algorithm," Comput. Methods Appl. Mech. Eng., vol. 376, no. January, p. 113609, 2021, doi: 10.1016/j.cma.2020.113609.

[15] R. Zheng, H. Jia, L. Abualigah, Q. Liu, and S. Wang, "An improved arithmetic optimization algorithm with forced switching mechanism for global optimization problems," Math. Biosci. Eng., vol. 19, no. 1, pp. 473–512, 2022, doi: 10.3934/mbe.2022023.

[16] F. A. Hashim, E. H. Houssein, K. Hussain, and M. S. Mabrouk, "ScienceDirect Honey Badger Algorithm : New metaheuristic algorithm for solving optimization problems," *Math. Comput. Simul.*, vol. 192, pp. 84–110, 2022, doi: 10.1016/j.matcom.2021.08.013.

[17] W. Elsayed, "Enhanced Honey Badger Algorithm With Lévy Flights for Solving Economic Load Dispatch Problems," *Eng. Res. J. - Fac. Eng.*, vol. 51, no. 4, pp. 19–29, 2022, doi: 10.21608/erjsh.2022.149753.1060.