New Ratio and Product Type Estimators for Population Mean Under Ranked Set Sampling

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Abstract

The study deals with the concept of Ranked Set Sampling (RSS) as an effective measure to find out the mean of the population when a sample of small size can be found without measuring or with other methods like ranking. This study has introduced modified ratio and product estimators to see the population mean under ranked set sampling and compare them with the existing ones in terms of Mean Square Error (MSE) and efficiency. The proposed estimators are better than the existing estimators like the mean per unit estimator, ratio estimator, and product estimator in ranked set sampling. Two tables are used to show the comparison between the proposed estimator and the existing estimators by using bivariate normal distribution. Empirical studies using simulations further validate the effectiveness of these new estimators, suggesting their potential for broader application in statistical analysis.

Keywords: Estimators, Population, Mean Square Error, Efficiency, Ranked Set Sampling

1. Introduction

The Ranked Set Sampling approach is utilized in situations in which it is difficult to measure all of the units that are included in the population, but it is simple and inexpensive to choose units that are included in a population of a smaller size. In [1] considered RSS as an example of a sampling design for which the inclusion probabilities are not equal and suggested that the sample mean is an unbiased estimator for the population mean. Estimation of population mean using exponential and product type estimator is given in [2] for ranked set sampling. Estimation of population distribution function under stratified random sampling given in [3]. In [4] RSS literature appear in two survey articles. Estimation of the population distribution function in simple random sampling using auxiliary variable is given in [5]. In case of measurement errors, it was given in [6]. Simulation and diverse application of RSS introduced in [7]. In [8], showed that the efficiency of the estimator depends upon the population and magnitude of errors in ranking. A new linear combination of ratio estimators under ranked set sampling given in [9] and [10] to estimate the mean of the population. Another design of ranked set sampling called dual sampling can be used to calculate the population distribution function given in [11].

2. Methodology

Let ($y_{j[1]}$,$x_{j[1]}$), ($y_{j[2]}$,$x_{j[2]}$),….., ($y_{j[m]}$,$x_{j[m]}$); j = 1, 2, ..., r be the ranked set sample obtained from the j^{th} replication process, where $y_{j[i]}$ denotes the i^{th} judgment order statistics for the variable y.

Some notations and basic results are as follows: Assume that $\mu_x$and μ_y be the population means of auxiliary and study variables respectively, $\bar{x}$ and $\bar{y}$ denotes the sample mean of auxiliary and study variables variable, correspondingly σ_x and σ_y denotes the population standard deviation. The mean per unit ranked set sample estimator $\bar{y}_{rss}$ is simply the average of the sample observation, given by the formula

$$\bar{y}_{rss} = \sum_{j=1}^{r} \sum_{i=1}^{m} \frac{y_{j(i)}}{rm}$$

And its variance is given by

$$V(\bar{y}_{rss}) = \frac{{\sigma_y}^2}{rm} - \frac{1}{r}\frac{1}{m^2} \sum_{i=1}^{m} (\mu_{y(i)} - \mu_y)^2$$

Also, we are having the following notations

$\mu_{x(i)} = E\,(x_{j[i]})$ $\qquad$ $\mu_{y(i)} = E\,(y_{j[i]})$

$\sigma^2_{x(i)} = V\,(x_{j[i]}) = E\,(x_{j[i]} - \mu_{x(i)})^2$ $\qquad$ $\sigma^2_{y(i)} = V\,(y_{j[i]}) = E\,(y_{j[i]} - \mu_{y(i)})^2$

$\tau_{x(i)} = \mu_{x(i)} - \mu_x$ $\qquad$ $\tau_{y(i)} = \mu_{y(i)} - \mu_y$

$\tau_{xy(i)} = \tau_{x(i)}\,\tau_{y(i)}$ $\qquad$ $\sigma_{xy[i]} = E\,(x_{j[i]} - \mu_{x(i)})\,(y_{j[i]} - \mu_{y(i)})$

Also, we can easily verify the following results

$\sum_{i=1}^{m} \mu_{x(i)} = m\mu_x$ $\qquad$ $\sum_{i=1}^{m} \mu_{y(i)} = m\mu_y$

$\sum_{i=1}^{m} \tau_{x(i)} = 0$ $\qquad$ $\sum_{i=1}^{m} \tau_{y(i)} = 0$

$\sum_{i=1}^{m} \sigma^2_{x(i)} = m\sigma^2_x - \sum_{i=1}^{m} \tau^2_{x(i)}$ $\qquad$ $\sum_{i=1}^{m} \sigma^2_{y(i)} = m\sigma^2_y - \sum_{i=1}^{m} \tau^2_{y(i)}$

$\sum_{i=1}^{m} \sigma^2_{xy(i)} = m\sigma^2_{xy} - \sum_{i=1}^{m} \tau^2_{xy(i)}$

3. Existing Estimators

The existing estimators in RSS are ratio type estimator and product type estimator.

3.1 Ratio Estimator Under RSS

The ratio estimator under RSS is defined as

$$\bar{y}_{Rrss} = \bar{y}_{rss} \frac{\mu_x}{\bar{x}_{rss}}$$

its MSE is given by

$$MSE\,(\bar{y}_{Rrss}) = \frac{1}{rm}\,({\sigma_y}^2 + R^2{\sigma_x}^2 - 2R\sigma_{xy}) - \frac{1}{r}\frac{1}{m^2} \sum_{i=1}^{m} (\tau_{y(i)} - R\tau_{x(i)})^2$$

3.2 Product Estimator Under RSS

The product estimator under RSS is defined as

$$\bar{y}_{Prss} = \bar{y}_{rss} \frac{\bar{x}_{rss}}{\mu_x}$$

its MSE is given by

$$\text{MSE}\ (\bar{y}_{Prss}) = \frac{1}{rm}\ (\sigma_y{}^2 + R^2\sigma_x{}^2 + 2R\sigma_{xy}) - \frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)} + R\tau_{x(i)})^2$$

4. Proposed Estimator I

$$\bar{y}_{prop1} = \frac{\bar{y}_{rss}}{2}\ \left[\left(\frac{\bar{x}_{rss}}{\mu_x}\right)^{1+\alpha_0} + \left(\frac{\bar{x}_{rss}}{\mu_x}\right)^{-1+\alpha_0}\right]$$

Where α_0 is a real constant, $\bar{y}_{rss}$ and $\bar{x}_{rss}$ are sample means of study and auxiliary variables respectively under RSS.

To obtain the approximate expression for bias and MSE of the proposed estimator, we will write $\bar{y}_{rss}$ and $\bar{x}_{rss}$ in terms of δ's as

$\bar{y}_{rss} = \mu_y(1+\delta_1)$ $\qquad\qquad$ $\bar{x}_{rss} = \mu_x(1+\delta_2)$

$$\bar{y}_{prop1} = \frac{\mu_y(1+\delta_1)}{2}\left[\ \left(\frac{\mu_x(1+\delta_2)}{\mu_x}\right)^{1+\alpha_0} + \left(\frac{\mu_x(1+\delta_2)}{\mu_x}\right)^{\alpha_0-1}\right]$$

$$= \frac{\mu_y(1+\delta_1)}{2}\ [(1+\delta_2)^{1+\alpha_0} + ((1+\delta_2)^{(\alpha_0-1)}\ ]$$

$$= \frac{\mu_y(1+\delta_1)}{2}\ [(1+\delta_2)^{1+\alpha_0} + (1+\delta_2)^{-1+\alpha_0}\ ]$$

After solving and including the terms up to degree two in δ_1 and δ_2, we have

$$\bar{y}_{prop1} = \mu_y\ (1 + \delta_1 + \alpha_0\,\delta_2 + \frac{(\alpha_0{}^2 - \alpha_0 + 1)\delta_2{}^2}{2} + \alpha_0\,\delta_1\delta_2)$$

$$\bar{y}_{prop1} - \mu_y = \mu_y\ \ (\delta_1 + \alpha_0\,\delta_2 + \frac{(\alpha_0{}^2 - \alpha_0 + 1)\delta_2{}^2}{2} + \alpha_0\,\delta_1\delta_2) \qquad (1)$$

After taking the expectation on both sides, we can obtain the expression for bias up to the first-order approximation as

$$\text{Bias}\ (\bar{y}_{prop1}) = \frac{1}{\mu_y}\ [\frac{(\alpha_0{}^2 - \alpha_0 + 1)}{2}\ R^2\ V\ (\bar{x}_{rss}) + \alpha_0\ R\ \text{Cov}\ (\bar{y}_{rss}\ , \bar{x}_{rss})]$$

Where $R = \frac{\mu_y}{\mu_x}$

Now substituting the values of $V(\bar{x}_{rss})$ and $Cov(\bar{y}_{rss}\ , \bar{x}_{rss})$, we get

$$\text{Bias}\ \ (\bar{y}_{prop1})\ = \frac{1}{rm}\frac{1}{\mu_y}\ [\frac{(\alpha_0{}^2 - \alpha_0 + 1)}{2}\ R^2\sigma_x{}^2 + \alpha_0\,R\sigma_{xy}\ ] - \frac{1}{r}\frac{1}{m^2}\frac{1}{\mu_y}\ [\frac{(\alpha_0{}^2 - \alpha_0 + 1)}{2}\ R^2\ \sum_{i=1}^{m}\tau_{x(i)}^2 + \alpha_0\,R \sum_{i=1}^{m}\tau_{xy(i)}\ ]$$

To find out the approximate expression for MSE, first, take square on both sides of (1) and consider the terms of δ's up to degree two

$$(\bar{y}_{prop1} - \mu_y)^2 = \mu_y^2(\delta_1^{\,2} + \alpha_0^{\,2}\delta_2^{\,2} + 2\alpha_0\,\delta_1\delta_2)$$

Taking expectation on both sides, we get

$$\text{MSE}\,(\bar{y}_{prop1}) = \text{V}(\bar{y}_{rss}) + \alpha_0^{\,2}R^2\,\text{V}\,(\bar{x}_{rss}) + 2\text{R}\,\alpha_0\,\text{Cov}(\bar{y}_{rss}\,,\bar{x}_{rss})$$

$$= \frac{1}{rm}\,(\sigma_y^{\,2} + \alpha_0^{\,2}R^2\sigma_x^{\,2} + 2\text{R}\alpha_0\,\sigma_{xy}) - \frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)} + \alpha_0\,R\tau_{x(i)})^2$$

Which is the required approximate expression for MSE

To find the minimum value of MSE, we will solve the equation

$$\frac{\partial MSE(\bar{y}_{prop1})}{\alpha_0} = 0$$

After solving we get the optimum value of α_0 as

$$\text{Optimal }(\alpha_0) = \frac{-\text{Cov}(\bar{y}_{rss}\,,\bar{x}_{rss})}{RV\,(\bar{x}_{rss})}$$

Now substituting this value in the expression of MSE, we get

$$\text{Minimum [MSE }(\bar{y}_{prop1})] = [1 - \rho^2(\bar{y}_{rss}\,,\bar{x}_{rss})]\,\text{V}(\bar{y}_{rss})$$

Where $\rho^2(\bar{y}_{rss}\,,\bar{x}_{rss}) = \dfrac{\text{Cov}^2(\bar{y}_{rss}\,,\bar{x}_{rss})}{\text{V}\,(\bar{x}_{rss})\text{V}(\bar{y}_{rss})}$

Case 1: When $\alpha_0 = 1$

$$\bar{y}_{prop1} = \frac{\bar{y}_{rss}}{2}\left[\left(\frac{\bar{x}_{rss}}{\mu_x}\right)^2 + 1\right]$$

Which is the average of the mean per unit and quadratic product type estimator in the RSS

Case 2: when $\alpha_0 = 0$

$$\bar{y}_{prop1} = \bar{y}_{rss}\left[\frac{\bar{x}_{rss}}{\mu_x} + \frac{\mu_x}{\bar{x}_{rss}}\right]$$

Which is the average of product and ratio type estimator in RSS

Case 3: When $\alpha_0 = -1$

$$\bar{y}_{prop1} = \frac{\bar{y}_{rss}}{2}\left[1 + \left(\frac{\mu_x}{\bar{x}_{rss}}\right)^2\right]$$

Which is the average of the mean per unit and quadratic ratio type estimator in the RSS

Comparison with $\bar{y}_{Rrss}$

$$\text{MSE }(\bar{y}_{Rrss}) - \text{Minimum [MSE }(\bar{y}_{prop1})] = [(\rho(\bar{y}_{rss}\,,\bar{x}_{rss})\sqrt{\text{V}(\bar{y}_{rss})} - R\sqrt{\text{V}\,(\bar{x}_{rss})}\,]^2 \geq 0$$

Which shows that $\bar{y}_{prop1}$ is always more efficient than $\bar{y}_{Rrss}$

Comparison with $\bar{y}_{Prss}$

MSE ($\bar{y}_{Prss}$) – Minimum [MSE ($\bar{y}_{prop1}$)] = $[(\rho(\bar{y}_{rss}, \bar{x}_{rss})\sqrt{V(\bar{y}_{rss})} + R\sqrt{V(\bar{x}_{rss})}]^2 \geq 0$

Which shows that $\bar{y}_{prop1}$ is always more efficient than $\bar{y}_{Prss}$

Sinha AK[12] and Alam, R., Hanif, M., Shahbaz, S.H. *et al*[13] also showed that the suggested estimators are better than the ratio and product type estimators.

5. Proposed Estimator II

$$\bar{y}_{prop2} = \frac{\bar{y}_{rss}}{2} \left[\left(\frac{\mu_x}{\bar{x}_{rss}}\right)^{1+\alpha_0} + \left(\frac{\mu_x}{\bar{x}_{rss}}\right)^{-1+\alpha_0} \right]$$

Where α_0 is a real constant, $\bar{y}_{rss}$ and $\bar{x}_{rss}$ are sample means of study and auxiliary variables respectively under RSS.

To obtain the approximate expression for bias and MSE of the proposed estimator, we will write $\bar{y}_{rss}$ and $\bar{x}_{rss}$ in terms of δ's as

$\bar{y}_{rss} = \mu_y(1+\delta_1)$ $\quad\quad$ $\bar{x}_{rss} = \mu_x(1+\delta_2)$

$$\bar{y}_{prop2} = \frac{\mu_y(1+\delta_1)}{2} \left[\left(\frac{\mu_x}{\mu_x(1+\delta_2)}\right)^{1+\alpha_0} + \left(\frac{\mu_x}{\mu_x(1+\delta_2)}\right)^{\alpha_0-1} \right]$$

$$= \frac{\mu_y(1+\delta_1)}{2} [(1+\delta_2)^{-1-\alpha_0} + ((1+\delta_2)^{-\alpha_0+1}]$$

After solving and including the terms up to degree two in δ_1 and δ_2, we have

$$\bar{y}_{prop2} = \mu_y \left(1 + \delta_1 - \alpha_0\, \delta_2 + \frac{({\alpha_0}^2+\alpha_0+1){\delta_2}^2}{2} - \alpha_0\, \delta_1 \delta_2\right)$$

$$\bar{y}_{prop2} - \mu_y = \mu_y \left(\delta_1 - \alpha_0\, \delta_2 + \frac{({\alpha_0}^2+\alpha_0+1){\delta_2}^2}{2} - \alpha_0\, \delta_1 \delta_2\right) \quad (2)$$

After taking the expectation on both sides, we can obtain the expression for bias up to the first-order approximation as

$$\text{Bias}(\bar{y}_{prop2}) = \frac{1}{\mu_y} \left[\frac{({\alpha_0}^2+\alpha_0+1)}{2} R^2\, V(\bar{x}_{rss}) - \alpha_0\, R\, \text{Cov}(\bar{y}_{rss}, \bar{x}_{rss})\right]$$

Where $R = \frac{\mu_y}{\mu_x}$

Now substituting the values of $V(\bar{x}_{rss})$ and $\text{Cov}(\bar{y}_{rss}, \bar{x}_{rss})$, we get

$$\text{Bias}(\bar{y}_{prop2}) = \frac{1}{rm}\frac{1}{\mu_y} \left[\frac{({\alpha_0}^2+\alpha_0+1)}{2} R^2 {\sigma_x}^2 - \alpha_0 R\sigma_{xy}\right] - \frac{1}{r}\frac{1}{m^2}\frac{1}{\mu_y} \left[\frac{({\alpha_0}^2+\alpha_0+1)}{2} R^2 \sum_{i=1}^{m} \tau_{x(i)}^2 - \alpha_0 R \sum_{i=1}^{m} \tau_{xy(i)}\right]$$

To find out the approximate expression for MSE, first take square on both sides of (2) and consider the terms of δ's up to degree two

$(\bar{y}_{prop2} - \mu_y)^2 = \mu_y{}^2(\delta_1{}^2 + \alpha_0{}^2\delta_2{}^2 - 2\alpha_0\,\delta_1\delta_2)$

Taking expectation on both sides, we get

$$\text{MSE}\,(\bar{y}_{prop2}) = \text{V}(\bar{y}_{rss}) + \alpha_0{}^2R^2\,\text{V}\,(\bar{x}_{rss}) - 2\text{R}\,\alpha_0\,\text{Cov}(\bar{y}_{rss}\,,\bar{x}_{rss})$$

$$= \frac{1}{rm}\,(\sigma_y{}^2 + \alpha_0{}^2R^2\sigma_x{}^2 - 2\text{R}\alpha_0\,\sigma_{xy}) - \frac{1}{r}\frac{1}{m^2}\sum_{i=1}^{m}(\tau_{y(i)} - \alpha_0\,R\tau_{x(i)})^2$$

Which is the required approximate expression for MSE

To find the minimum value of MSE, we will solve the equation

$$\frac{\partial MSE(\bar{y}_{prop2})}{\alpha_0} = 0$$

After solving we get optimum value of α_0 as

$$\text{Optimal } (\alpha_0) = \frac{\text{Cov}(\bar{y}_{rss}\,,\bar{x}_{rss})}{RV\,(\bar{x}_{rss})}$$

Now substituting this value in the expression of MSE, we get

$$\text{Minimum [MSE } (\bar{y}_{prop2})] = [1 - \rho^2(\bar{y}_{rss}\,,\bar{x}_{rss})]\,\text{V}(\bar{y}_{rss})$$

Where $\rho^2(\bar{y}_{rss}\,,\bar{x}_{rss}) = \dfrac{\text{Cov}^2(\bar{y}_{rss}\,,\bar{x}_{rss})}{\text{V}\,(\bar{x}_{rss})\text{V}(\bar{y}_{rss})}$

Case 1: When $\alpha_0 = 1$

$$\bar{y}_{prop2} = \frac{\bar{y}_{rss}}{2}\left[\left(\frac{\mu_x}{\bar{x}_{rss}}\right)^2 + 1\right]$$

Which is the average of the mean per unit and quadratic ratio type estimator in the RSS

Case 2: when $\alpha_0 = 0$

$$\bar{y}_{prop2} = \bar{y}_{rss}\left[\frac{\bar{x}_{rss}}{\mu_x} + \frac{\mu_x}{\bar{x}_{rss}}\right]$$

Which is the average of product and ratio type estimator in RSS

Case 3: When $\alpha_0 = -1$

$$\bar{y}_{prop2} = \frac{\bar{y}_{rss}}{2}\left[1 + \left(\frac{\bar{x}_{rss}}{\mu_x}\right)^2\right]$$

Which is the average of the mean per unit and quadratic product type estimator in the RSS

Comparison with $\bar{y}_{Rrss}$

$$\text{MSE}\,(\bar{y}_{Rrss}) - \text{Minimum [MSE } (\bar{y}_{prop2})] = [(\rho(\bar{y}_{rss}\,,\bar{x}_{rss})\sqrt{\text{V}(\bar{y}_{rss})} - R\sqrt{\text{V}\,(\bar{x}_{rss})}]^2 \geq 0$$

Which shows that $\bar{y}_{prop2}$ is always more efficient than $\bar{y}_{Rrss}$

Comparison with $\bar{y}_{Prss}$

MSE ($\bar{y}_{Prss}$) – Minimum [MSE ($\bar{y}_{prop2}$)] = $[(\rho(\bar{y}_{rss}, \bar{x}_{rss})\sqrt{V(\bar{y}_{rss})} + R\sqrt{V(\bar{x}_{rss})}]^2 \geq 0$

Which shows that $\bar{y}_{prop2}$ is always more efficient than $\bar{y}_{Prss}$

6. Result and Discussion

We perform a simulation study to verify the results of the proposed estimators. A total number of 1,000 samples are drawn from bivariate normal distribution BVN (200, 100, 4, 4, ρ) for ρ = -0.7, -0.5, -0.3, 0.3, 0.5, 0.7 under RSS. The efficiency of an estimator Δ with respect to $\bar{y}_{rss}$ to estimate the population mean is

Efficiency (Δ) = $\frac{MSE(\bar{y}_{rss})}{MSE(\Delta)}$

The MSE values of $\bar{y}_{rss}$, $\bar{y}_{Rrss}$, $\bar{y}_{Prss}$, $\bar{y}_{prop1}$, $\bar{y}_{prop2}$ are obtained for different values of n, m, and r in table 1.

The efficiencies of $\bar{y}_{Rrss}$, $\bar{y}_{Prss}$, $\bar{y}_{prop1}$, $\bar{y}_{prop2}$ with respect to $\bar{y}_{rss}$ for different values of ρ and n are shown in table 2.

Table 1: MSE (n = 12, m = 4, r = 3)

ρ	MSE ($\bar{y}_{rss}$)	MSE ($\bar{y}_{Rrss}$)	MSE ($\bar{y}_{Prss}$)	MSE ($\bar{y}_{prop1}$)	MSE ($\bar{y}_{prop2}$)
-0.7	0.33475	1.48903	0.70457	0.09962	0.11618
-0.5	0.33475	1.56394	1.00284	0.12259	0.14332
-0.3	0.33475	1.57367	1.23878	0.13417	0.15696
0.3	0.33475	1.23961	1.57318	0.13428	0.15716
0.5	0.33475	1.00288	1.56033	0.12277	0.14350
0.7	0.33475	0.70402	1.48545	0.09982	0.11643

Table 2: Efficiencies

ρ	Eff ($\bar{y}_{rss}$)	Eff ($\bar{y}_{Rrss}$)	Eff ($\bar{y}_{Prss}$)	Eff ($\bar{y}_{prop1}$)	Eff ($\bar{y}_{prop2}$)
-0.7	1	0.22481	0.47512	3.36036	2.88125
-0.5	1	0.21405	0.33381	2.73071	2.33571
-0.3	1	0.21236	0.27023	2.49507	2.13270

0.3	1	0.27005	0.21279	2.49289	2.12999
0.5	1	0.33379	0.21454	2.72672	2.33272
0.7	1	0.47549	0.22536	3.35365	2.87525

It is clear from table 1 and 2 that proposed estimators have less mean square error and higher efficiency than the existing estimators. It is also seen that for different values of α_0 the proposed estimators reduce to average of some existing estimators. Also, the optimum value of α_0 give minimum mean square error which is the same in both the proposed estimators. Our findings indicate that the proposed estimators provide more efficient estimates under certain conditions, particularly when sample sizes are small and rankings are either perfect or imperfect.

7. Conclusion

The proposed estimator (1) $\bar{y}_{prop1}$ and proposed estimator (2) $\bar{y}_{prop2}$ is found out to be more statistically efficient than the mean per unit estimator, product estimator, and ratio estimator in RSS. It can be calculated that the MSE and efficiencies for different values of n, r, and m. But in all the cases it can be seen that the proposed estimators are much better than the existing estimators. As we increased the sample size, the efficiency of the proposed estimator was found to be higher. In the same way as ρ increases, the efficiency of the proposed estimator also increases. Hence the use of the proposed estimator of population mean would be more beneficial than the existing ones.

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