Motion of Strong Self-gravitating Shock in weak Magnetic field

[1]Pushpender Kumar Gangwar, [2]Rajesh Kumar Verma, [3]Yogember Singh [4]Deepak Kumar

[1]Assistant Professor, Department of Physics, Bareilly College, Bareilly, India
[2]Assistant Professor, Department of Physics, K.S. Saket (P.G.) College, Ayodhya, India
[3]Professor, Department of Physics, K.G.K. (P.G.) College, Moradabad, India
[4]Assistant Professor, Department of Physics, S. L. Education Institute, Moradabad, India

[1]dr.pkgangwar@gmail.com, [2]iitr.rajesh@gmail.com, [3]dr.yogembersingh006@gmail.com, [4]dkthakurpms@gmail.com

***Abstract*—In the present work, the effect of weak magnetic field on motion of strong converging shock wave in an ideal gas having a power varying density distribution has been carried out. The magnetic field is taken to be axial and initially of constant strength. Neglecting the effect of overtaking disturbances, the CCW method has been applied to the solution of the problem. The analytical expressions for shock strength, shock velocity, pressure, and particle velocity have been derived. The variations of all flow variables with the convergence of shock front, adiabatic index, and density parameter have been numerically computed and discussed through graphs and table. Finally, the results obtained here have been compared with those for diverging shock as well as with earlier results.**

Key words— CCW method, self-gravitation, ideal gas, weak magnetic field.

I. Introduction

On account of the importance of weak shock wave in astrophysics, the study of the motion of strong converging shock wave in the presence of a magnetic field has received considerable attention from several scientists. In recent years, several methods and techniques have been used to study the propagation of strong shock in self-gravitating gas under the effect of magnetic field. Pai[1], [2] Prakash and Kumar [3], and many others authors applied the similarity method to explore the shock propagation problem with and without overtaking effect behind the front. Kumar and Saxena [4] have discussed the propagation of hydromagnetic cylindrical shock wave through self-gravitating gas with different type of initial density distributions. Considering the effect of overtaking disturbances on the motion of hydromagnetic cylindrical and spherical diverging shock waves in self-gravitating gas has been studied by Yadav et al [5], [6], [7]. Gangwar [8] have investigated the production of entropy on adiabatic propagation of strong spherical imploding shock waves in a dusty gaseous atmosphere having solid body rotation, including the influence of overtaking shock, by using CCW theory.

In this work, the effect of the weak magnetic field, the adiabatic propagation of strong hydrodynamic converging shock through self-gravitating gas having an initial density distribution $\rho_0=\rho' r^{-\omega}$ where ρ' is the density at the centre and ω is a constant, has been studied by Chester[9]-Chesnell[10]-Witham[11] method. The magnetic field is taken to be axial and initially of constant strength. Neglecting the effect of overtaking disturbances, the analytical expressions for shock strength, shock velocity, pressure and particle velocity have been derived. Dependence of these parameters with propagation distance (r), adiabatic index (γ) density parameter (ω), and constant (ξ) has been numerically calculated and discussed through graphs and table.

Finally, the results obtained here have been compared with those for the diverging shock Yadav et al. [7] as well as with earlier results and Yadav Gangwar[12].

II. Governing equations, jump conditions and analytical expressions

Under the influence of its own gravitation and an axial magnetic field, the basic equations for the flow behind the spherical shock wave are written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{2\rho}\frac{\partial H^2}{\partial r}+\frac{Gm}{r^2}=0 \tag{1}$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\left(p\rho^{-\gamma}\right)=0 \tag{3}$$

$$\frac{\partial H}{\partial t}+u\frac{\partial H}{\partial r}+H\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0 \tag{4}$$

$$\frac{\partial m}{\partial r}-4\pi r^2\rho=0 \tag{5}$$

where r is the radial co-ordinate, u, p, ρ, H and ω are respectively, particle velocity, the pressure, the density, the axial magnetic field, permeability of gas, m denotes the mass of the sphere of radius r, γ is the adiabatic index of gas and G is the gravitational constant.

The magenetohydrodynamic shock boundary conditions can be written in terms of a single parameter $\xi=\rho/\rho_0$ as

$$\rho = \rho_0 \xi, \qquad H = H_0 \xi, \quad u = \frac{(\xi - 1)}{\xi} U \tag{6}$$

$$U^2 = \frac{2\xi}{(\gamma+1) - (\gamma-1)\xi}\left[a_0^2 + \frac{b_0^2}{2}\{(2-\gamma)\xi + \gamma\}\right] \tag{7}$$

$$p = p_0 + \frac{2\rho_0(\xi - 1)}{(\gamma+1) - (\gamma-1)\xi}\left[a_0^2 + \frac{(\gamma-1)}{4} b_0^2 (\xi - 1)^2\right] \tag{8}$$

where suffix "o" stands for the state immediately ahead of the shock, a_0 is the speed of sound ($\sqrt{\gamma p_0/\rho_0}$), U is the shock velocity, in an unperturbed medium and b_0 is the Alfven speed ($\sqrt{\mu H_0^2/\rho_0}$). The non-dimensional pressure p/p_0 is large in the case of strong shock.

For weak magnetic field $\left(b_0^2 << a_0^2\right)$ the jump conditions across the shock front (6)-(8) is reduce to

$$\rho = \rho_0 \xi, \qquad H = H_0 \xi, \quad u = \frac{(\xi - 1)}{\xi} U \tag{9}$$

$$\frac{p}{p_0} = 1 + \left(\chi' a_0^2 + N_1 b_0^2\right)\frac{U^2}{a_0^4} \tag{10}$$

$$\chi' = \frac{\gamma(\xi-1)}{\xi}, \qquad N_1 = \frac{\gamma(\xi-1)}{4\xi}\left[(\gamma-1)(\xi-1)^2 - 2\{(2-\gamma)\xi + \gamma\}\right] \tag{11}$$

For spherical converging shocks, the characteristic form of the system of equations (1)-(4), i.e. the form in which each equation contains derivatives in only one direction in (r, t) plane, is

$$dp - \rho c du + \mu H dH + \frac{2\rho c^2 u}{u-c}\frac{dr}{r} - \frac{\rho c G m}{u-c}\frac{dr}{r^2} = 0 \tag{12}$$

Here $c^2 = a^2 + b^2 = \left(\gamma p + \mu H^2\right)/\rho$.

Propagation of strong spherical shock can be analyzed by substituting conditions(9)-(11) and $c = U\sqrt{\chi'/\xi}$ into equation (12), at H_o= constant, we get. The condition of hydrostatic equilibrium prevailing the front of the shock is written by substituting $\partial/\partial t = 0 = u$ and, H_o is constant, in equation (1), we get

$$\frac{1}{\rho_0}\frac{dp_0}{dr} + \frac{Gm}{r^2} = 0 \tag{13}$$

Assuming that the medium in which shock propagates is at rest and initial density distribution is given by

$$\rho_0 = \rho' r^{-\omega} \tag{14}$$

where, where ρ' is the density at the centre and ω is a constant. The mass m can be written with the use of equation (5) and (14), we get

$$\Rightarrow \qquad m = \frac{4\pi\rho' r^{3-\omega}}{3-\omega} \tag{15}$$

Substituting the value of ρ_o and m from equation (14) and (15), respectively, in equation (13), we get

$$dp_0 = -\frac{4\pi\rho'^2 G r^{1-2\omega}}{(3-\omega)} dr \tag{16}$$

On integrating above equation, we get

$$p_0 = A\rho'^2 G r^{2(1-\omega)} \tag{17}$$

where $A = \frac{2\pi}{(\omega-1)(3-\omega)}$

$$\frac{dp_0}{p_0} = -\frac{4\pi\rho'^2 G r^{1-2\omega}(\omega-1)(3-\omega)}{2\pi\rho' G r^{2(1-\omega)}(3-\omega)} dr = 2(1-\omega)\frac{dr}{r} \tag{18}$$

For positive finite pressure, as defined by equation (6.15), requires that ω should obey the inequality as $1 < \omega < 3$ Using equation(17) and (14), we get

$$a_0^2 = \gamma A \rho' G r^{(2-\omega)} \tag{19}$$

Or

$$\frac{da_0}{a_0} = \frac{(2-\omega)}{2} r^{(1-\omega)} \tag{20}$$

We have

$$b_0^2 = \frac{\mu H_0^2}{\rho'} r^{-\omega} \Rightarrow \frac{db_0}{b_0} = \frac{\omega}{2}\frac{dr}{r} \tag{21}$$

Substituting the values of respective quantities from(14), (15)-(18) and (21) in equation**Error! Reference source not found.**, and putting $\beta^2 = \mu H_0^2 / \gamma p'$ and $\gamma p' = \rho' a'^2$ after simplification, we get

$$dU^2 + D_1 U^2 \frac{dr}{r} + \frac{D_2 U^2 \beta^2 r^{2\omega-3}}{\rho' G} dr - D_3 \rho' G r^{1-\omega} dr + D_4 \beta^2 r^{\omega-1} dr = 0 \tag{22}$$

where $D_1 = \frac{\chi'}{N_2}\left[\frac{2(\xi-1)}{\left\{(\xi-1)-\sqrt{\chi'\xi}\right\}} - \frac{\omega}{\gamma}\right]$

$$D_2 = \frac{N_1 a'^2}{A N_2 \gamma^2}\left[\omega - 2 - D_1\right]$$

$$D_3 = \frac{2}{N_2}\left[\frac{2\pi\xi\sqrt{\chi'\xi}}{(3-\omega)\left\{(\xi-1)-\sqrt{\chi'\xi}\right\}} + A(\omega-1)\right]$$

$$D_4 = \frac{N_1 \chi' D_3}{A N_2 \gamma^2}$$

where $N_2 = \frac{\chi'}{\gamma} - \frac{(\xi-1)}{2}\sqrt{\frac{\chi'}{\xi}}$ and $\left[\frac{p_0 \chi' N_1}{N_2 \mu H_0^2} < 1\right]$

Integrating this equation, we have

$$U^2 = \left[k_2 r^{-D_1} + \frac{D_3 \rho' G r^{2-\omega}}{D_1 - \omega + 2} + \frac{D_2 D_3 \beta^2 r^{\omega}}{2(\omega-1)(D_1+\omega)} - \frac{D_4 \beta^2 r^{\omega}}{(D_1+\omega)}\right]^{1/2} \exp\left\{\frac{D_2 \beta^2 r^{2(\omega-1)}}{(1-\omega)\rho' G}\right\} \tag{23}$$

where k_2 is constant of integration.

Expressions for shock velocity [$U/\sqrt{\rho' G}$] and shock strength (U/a_o) for strong spherical converging shock in the presence of weak magnetic field are given by

$$\frac{U}{\sqrt{\rho' G}} = \left[\frac{k_1 r^{-D_1}}{\rho' G} + \frac{D_3 r^{2-\omega}}{D_1-\omega+2} + \frac{D_2 D_3 \beta^2 r^{\omega}}{2(\omega-1)(D_1+\omega)\rho' G} - \frac{D_4 \beta^{-2} r^{\omega}}{(D_1+\omega)\rho' G}\right]^{1/2} \exp\left\{\frac{D_2 \beta^2 r^{2(\omega-1)}}{4(1-\omega)\rho' G}\right\} \tag{24}$$

$$\frac{U}{a_0} = \frac{1}{\sqrt{\gamma A}}\left[\frac{k_2 r^{\omega-D_1-2}}{\rho' G} + \frac{D_3}{D_1-\omega+2} + \frac{D_2 D_3 \beta^2 \rho' G r^{2(\omega-1)}}{2(\omega-1)(D_1+\omega)} - \frac{D_4 \beta^2 r^{2(\omega-1)}}{(D_1+\omega)\rho' G}\right] \exp\left\{\frac{D_2 \beta^2 r^{2(\omega-1)}}{4(1-\omega)\rho' G}\right\} \tag{25}$$

III. Results and Discussion

Expressions(24) and (25), representing the shock velocity $[U/\sqrt{\rho' G}]$ and shock strength (U/a_o) for weak spherical converging shock in a self-gravitating gas in presence of weak magnetic field, shows its dependence on adiabatic index (γ), density parameter (ω), propagation distance (r), constant (ξ) and gravitation constant G. Taking the initial strength of shock $U/a_o = 1.25$ at $r = 2$, $\omega = 1.1$, $\xi = 9$ and $\beta^2 = 0.1$, for $\gamma = 1.4$, $D_2\beta^2 r^{2(\omega-1)} / \{2(1-\omega)\rho' G\} = 0.25$, $a'^2/\rho' G = 0.9795$ and $k_2/\rho' G = 587.6281$, the dependence of shock velocity $[U/\sqrt{\rho' G}]$ and shock strength (U/a_o), pressure $(p/\rho'^2 G)$ and particle velocity $[u/\sqrt{\rho' G}]$ with propagation distance (r), ω,ξ and adiabatic index (γ) are numerically calculated and shown in figures (1-16)and Table 1, respectively.

Expressions for the pressure $(p/\rho'^2 G)$ and the particle velocity $[u/\sqrt{\rho' G}]$ immediately behind the strong spherical converging shock in presence of weak magnetic field, can be written as

$$\frac{p}{\rho'^2 G} = A r^{2(1-\omega)} + \frac{1}{\gamma}\left\{\chi' + \frac{C_1 a'^2 r^{2(\omega-1)}}{\gamma A \rho' G}\right\}\left[\frac{k_2 r^{-D_1} + \omega}{\rho' G} + \frac{D_3 r^{2(1-\omega)}}{D_1 - \omega + 2}\right.$$

$$\left. + \frac{D_2 D_3 \beta^2}{2(\omega-1)(D_1+\omega)\rho' G} - \frac{D_4 \beta^2}{(D_1+\omega)\rho' G}\right] \exp\left\{\frac{D_2 \beta^2 r^{2(\omega-2)}}{2(1-\omega)\rho' G}\right\} \tag{26}$$

$$\frac{u}{\sqrt{\rho' G}} = \frac{\xi-1}{\xi}\left[k_2 r^{-D_1} + \frac{D_3 \rho' G r^{2-\omega}}{D_1 - \omega + 2} + \frac{D_2 D_3 \beta^2 r^{\omega}}{2(\omega-1)(D_1+\omega)} - \frac{D_4 \beta^2 r^{\omega}}{(D_1+\omega)}\right]^{1/2}$$

$$\exp\left\{\frac{D_2 \beta^2 r^{2(\omega-1)}}{4(1-\omega)\rho' G}\right\} \tag{27}$$

It is found that the shock velocity $[U/\sqrt{\rho' G}]$ and shock strength (U/a_o) both increase as shock propagates [cf. Fig. (1-2)], whereas in case of diverging shock these parameters decrease as shock diverges [Yadav et al. (1995)]. Shock velocity and shock strength both increase as β^2 increases ω [cf. Fig. (3 and 4)].Shock velocity $[U/\sqrt{\rho' G}]$ decreases if ω increases [cf. Fig. (5)]. The dependence of shock strength on ω is same as in case of strong shock, initially, it increases with ω and after some value depending on γ , it decreases as ω increases [cf. Fig. (6)]. Shock velocity and shock strength both decrease with increase in γ [rf. Table 1]. It is clear from the figures (7) and (8), shock velocity and shock strength both decrease as ξ increases. The variation of the pressure $(p/\rho'^2 G)$ and particle velocity $[u/\sqrt{\rho' G}]$ with propagation distance (r) for ω = 1.1, 1.5 and 2.0 has been shown in figure (9) and (10), respectively. It is found that pressure and particle velocity both increase as shock converges. The pressure $(p/\rho'^2 G)$ and particle velocity $[u/\sqrt{\rho' G}]$ increase as β^2 increases [cf. Fig. (11 and 12)]. It is clear from the figure (13 and 14), respectively, that pressure and particle velocity both decrease with increase in ω. the pressure and particle velocity both decrease with increase in ξ [cf. Fig. (15 and 16)]. Table 1 shows that pressure and particle velocity both increase with γ.

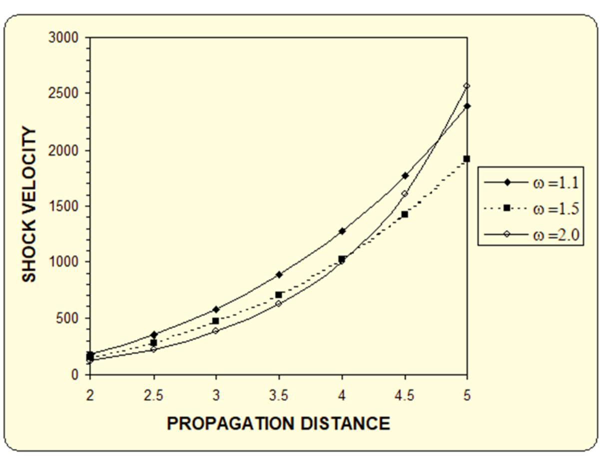

Figure 1: Variation of shock velocity $[U/\sqrt{\rho' G}]$ with propagation distance (r) at $\beta^2 = 0.1$, $\xi = 9$ and $\gamma = 1.4$.

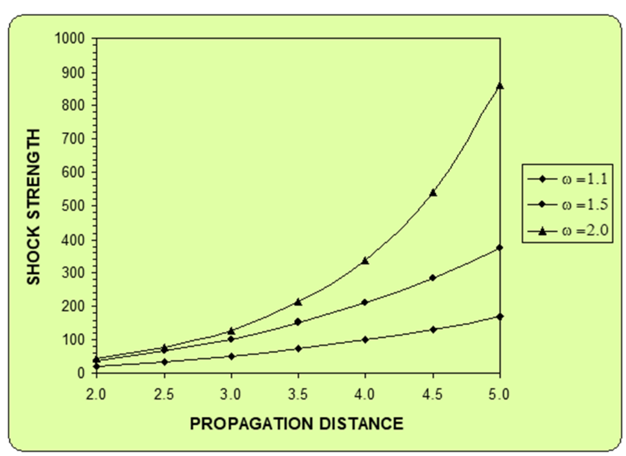

Figure 2: Variation of shock strength (U/a_o) with propagation distance (r) at $\beta^2 = 2$, $\xi = 9$ and $\gamma = 1.4$.

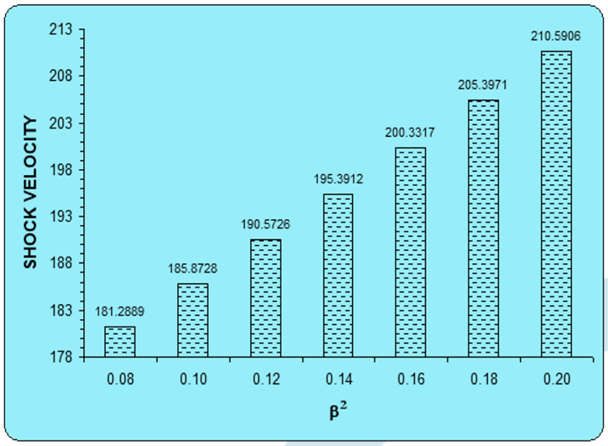

Figure 3: Variation of shock velocity [$U/\sqrt{\rho' G}$] with β^2 ω at r = 2, at ω = 1.1, ξ = 9 and γ = 1.4.

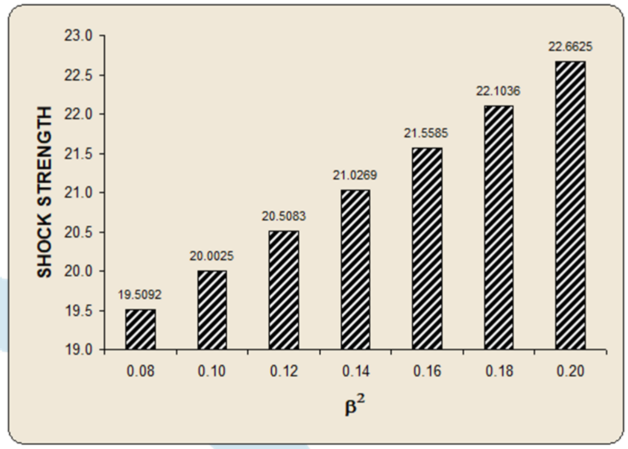

Figure 4: Variation of shock strength (U/a_o) with β^2 ω at r = 2, at ω = 1.1, ξ = 9 and γ = 1.4.

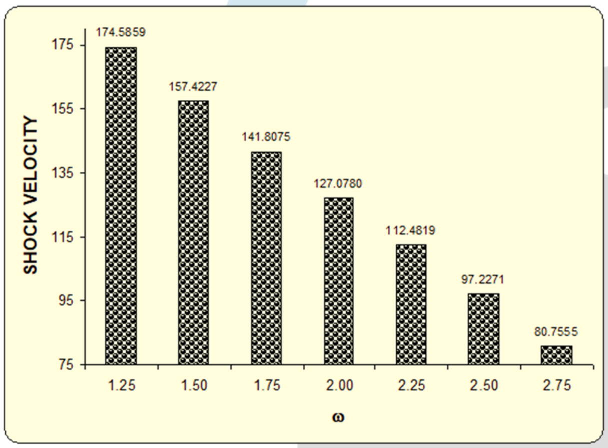

Figure 5: Variation of shock velocity [$U/\sqrt{\rho' G}$] with ω at β^2=0.1, r = 2, ξ = 1.5 and γ= 1.4

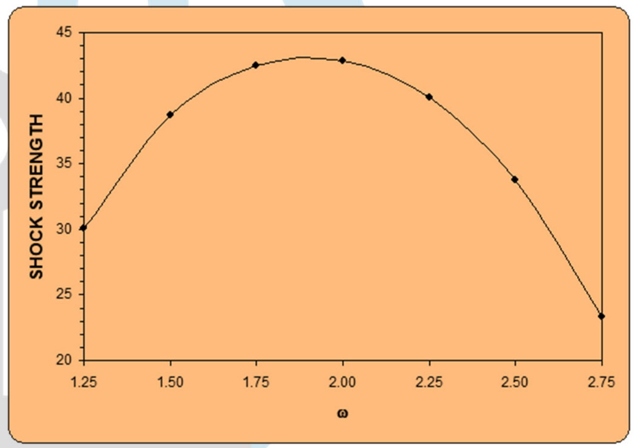

Figure 6: Variation of shock strength (U/a_o) with ω at β^2=0.1, r = 2, ξ = 1.5 and γ= 1.4

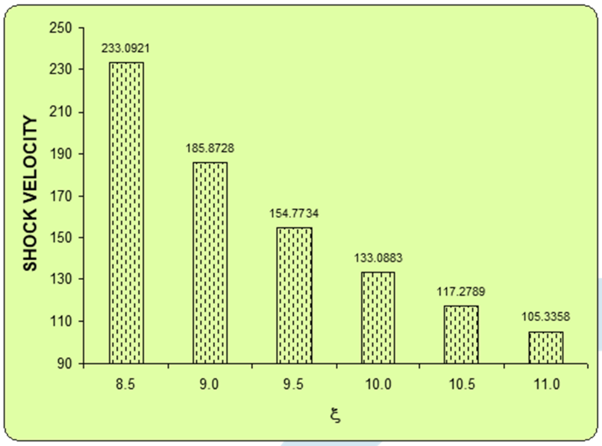

Figure 7: Variation of shock velocity $[U/\sqrt{\rho' G}]$ with ξ at ω=1.1, $\beta^2 = 0.1$, r = 2 and $\gamma = 1.4$

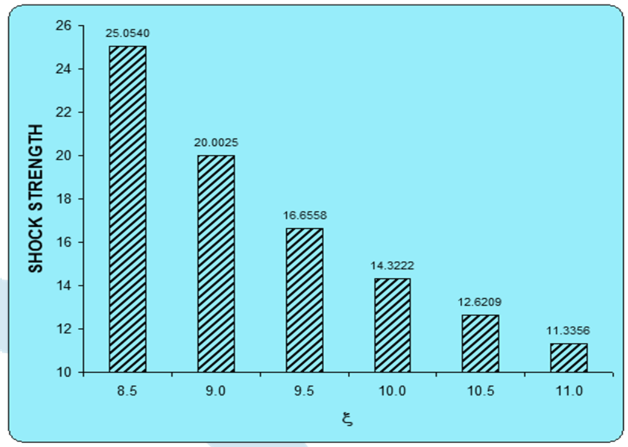

Figure 8: Variation of shock strength (U/a_o) with $[U/\sqrt{\rho' G}]$ with ξ at ω=1.1, $\beta^2 = 0.1$, r = 2 and $\gamma = 1.4$

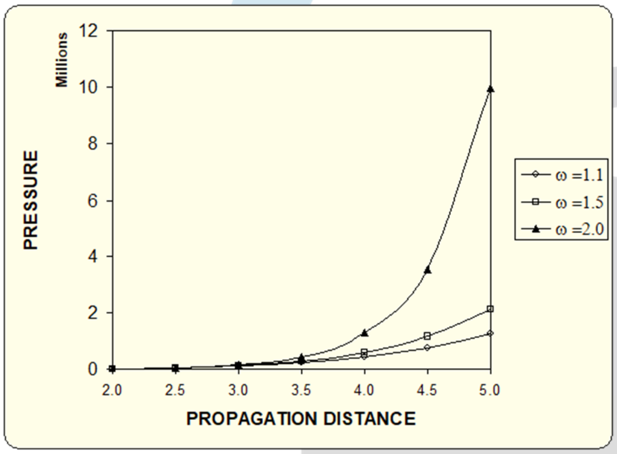

Figure 9: Variation of pressure (p/ρ'^2G) with propagation distance (r)

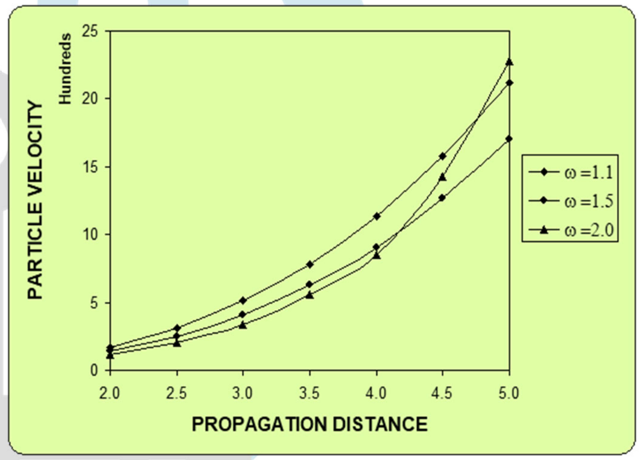

Figure 10: Variation of particle velocity $[u/\sqrt{\rho' G}]$ with propagation distance (r)

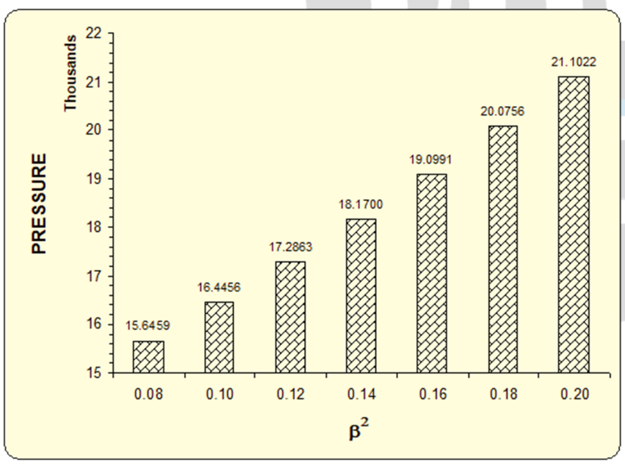

Figure 11 : Variation of pressure (p/ρ'^2G) with β^2 at ω= 1.1, r = 2 , ξ =9 and γ =1.4

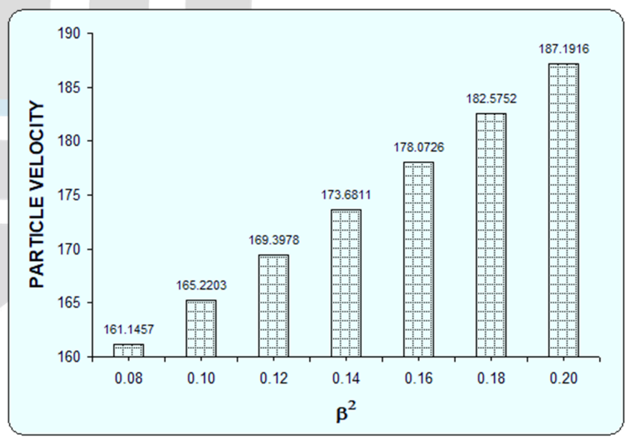

Figure 12: Variation of particle velocity $[u/\sqrt{\rho' G}]$ with β^2 at ω= 1.1, r = 2 , ξ =9 and γ =1.4

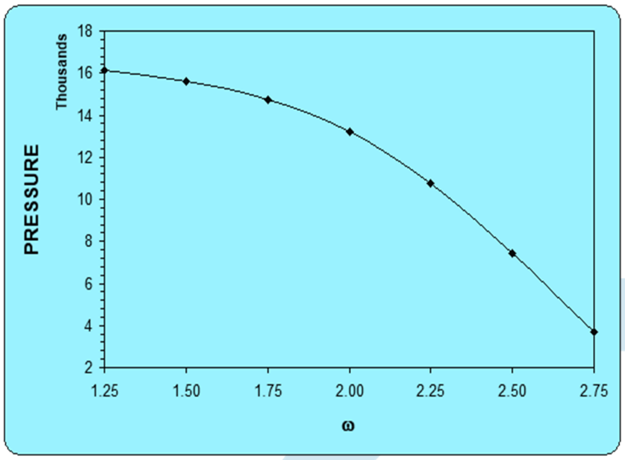

Figure 13: Variation of pressure (p/ρ'^2G) with ω at r = 2 , ξ =9, β^2 =0.1 and γ =1.4.

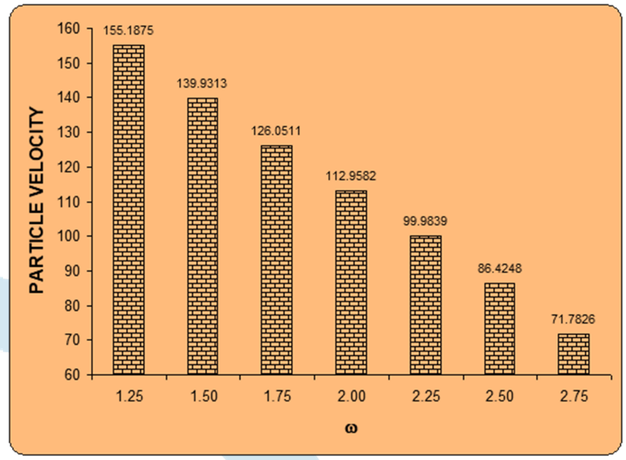

Figure 14: Variation of particle velocity $[\, u/\sqrt{\rho' G}\,]$ with ω at r = 2 , ξ =9, β^2 =0.1 and γ =1.4.

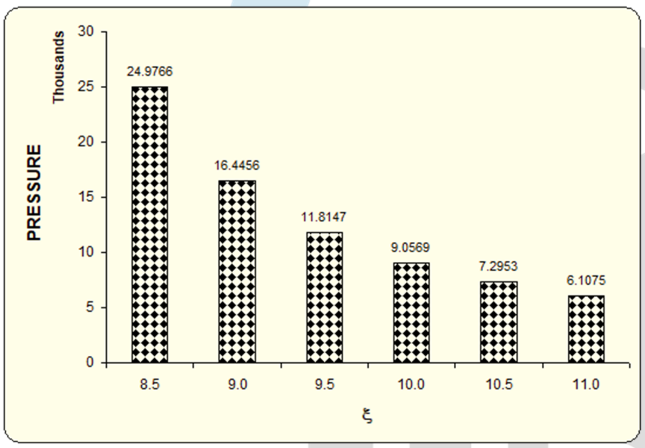

Figure 15: Variation of pressure (p/ρ'^2G) with ξ at ω= 1.1, β^2= 0.1, r = 2 and γ=1.4

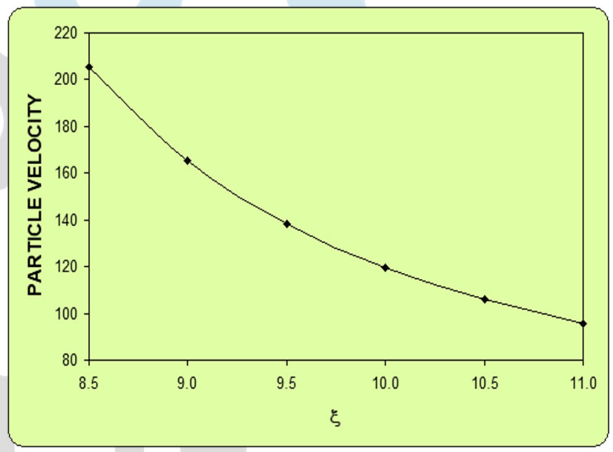

Figure 16: Variation of particle velocity $[\, u/\sqrt{\rho' G}\,]$ with ξ at ω= 1.1, β^2= 2, r = 2 and γ=1.4

Table 1: Variation of shock velocity ($U/\sqrt{\rho' G}$), shock strength (U/a_o), pressure (p/ρ'^2G) and particle velocity ($u/\sqrt{\rho' G}$) with adiabatic index (γ) at ξ = 1.1, β^2 = 2, ω = 1.5 and r = 2

Adiabatic index	Shock velocity	Shock strength	Pressure	Particle velocity
1.20	**135.9450**	**15.8018**	**7257.7076**	**120.8399**
1.40	**185.8728**	**20.0025**	**16445.6036**	**165.2203**
1.67	**626.9754**	**25.9114**	**38527.1920**	**233.7559**

REFERENCES

[1] S. I. Pai, "Cylindrical shock waves produced by instantaneous energy release in magneto- gas dynamics," United States, 1958. Available: https://www.osti.gov/biblio/4146431

[2] S. I. Pai, "Shock Wave Propagation in an Infinitely Electrically Conductive Gas with Transverse Magnetic Field and Gravitation," *ZAMM - J. Appl. Math. Mech. / Zeitschrift für Angew. Math. und Mech.*, vol. 39, no. 1–2, pp. 40–49, 1959, doi: https://doi.org/10.1002/zamm.19590390109.

[3] R. Praksh and S. Kumar, "Plane and cylindrical strong shocks in magnetogasdynamics," *Curr. Sci.*, vol. 56, no. 3, pp. 130–132, Jun. 1987. Available: http://www.jstor.org/stable/24091045

[4] S. Kumar and D. K. Saxena, "Hydromagnetic cylindrical shock in self-gravitating gas," *Astrophys. Space Sci.*, vol. 100, no. 1, pp. 65–83, 1984, doi: 10.1007/BF00651586.

[5] R. P. Yadav, M. Gupta, and S. Tripathi, "Effect of overtaking disturbances on strong cylindrical shock in a self-gravitating gas," *Shock Waves*, vol. 4, no. 3, pp. 163–165, 1994, doi: 10.1007/BF01417432.

[6] R. P. Yadav and S. Tripathi, "Effect of overtaking disturbances on the propagation of spherical shock wave through self-gravitating gas," no. 1991, pp. 67–80, 1995.

[7] R. P. Yadav, M. Gupta, and S. Kumar, "Effect of overtaking disturbances on a strong cylindrical hydromagnetic shock wave in a self-gravitating gas," *Astrophys. Space Sci.*, vol. 235, no. 1, pp. 27–39, 1996, doi: 10.1007/BF00643588.

[8] P. K. Gangwar, "Entropy production on dusty shock propagation in presence of overtaking disturbances," *AIP Conf. Proc.*, vol. 2220, no. May, 2020, doi: 10.1063/5.0001960.

[9] W. Chester, "The diffraction and reflection of shock waves," *Q. J. Mech. Appl. Math.*, vol. 7, no. 1, pp. 57–82, 1954, doi: 10.1093/qjmam/7.1.57.

[10] R. F. Chisnell, "The normal motion of a shock wave through a non-uniform one-dimensional medium," *Proc. R. Soc. London. Ser. A. Math. Phys. Sci.*, vol. 232, no. 1190, pp. 350–370, Nov. 1955, doi: 10.1098/RSPA.1955.0223.
[11] G. B. Whitham, "On the propagation of shock waves through regions of non-uniform area or flow," *J. Fluid Mech.*, vol. 4, no. 4, pp. 337–360, 1958, doi: 10.1017/S0022112058000495.
[12] R. P. Yadav and P. K. Gangwar, "Theoretical study of propagation of spherical converging shock waves in self gravitating gas," *Model. Meas. Control B*, vol. 72, no. 6, pp. 39–54, 2003.