A STUDY ON EVEN GRACEFUL LABELING FOR JEWEL GRAPH AND EXTENDED JEWEL GRAPH WITHOUT THE PRIME EDGE

[1]S. MOHAN, [2]V. S. AARTHI

[1]Assistant Professor, [2]Scholar
Department of Mathematics,
A.V.C College (Autonomous), Mannampandal, Mayiladuthurai, Tamilnadu.

Abstract

This Paper proposes the concept of Even Graceful Labeling in the Jewel Graph and the Extended Jewel Graph without the Prime edge.

Keywords Even Graceful Labeling, Jewel, Extended Jewel Graph.

1. INTRODUCTION

Graph theory is the study of graphs that are mathematical structures which can be used to model pairwise relation between objects. A graph is made up of vertices which are connected by edges[3]. A Graph labeling can be defined as an assignment ***f*** of labels to the vertices of ***G*** which induces for each edge xy a label depending on the vertex labels ***f(x)*** and ***f(y)***[1]. Graph labeling is a prominent area of research in Graph theory that has rigorous applications in coding theory, communication networks, Optimal circuit's layouts and graph decomposition problems[2].

The first graph labeling method known as the graceful labeling was introduced by *Rosa*[5]. The graceful labeling of a graph G with q edges is an injection f from the vertices of G to the set ***{0,1,2.....q}*** such that when each edge ***xy*** is assigned the label **|*f(x)-f(y)*|**, the resulting edge are distinct. In *2017*, *Elsonbaty* and *Daoud*[6] introduced new type of labeling called as even graceful labeling. The Even graceful labeling G with q edges is an injection ***f*: V(G)→ {0,1,2 .....2q-1}*** such that the edge ***f*: E(G)→ {2,4,6....2q-2}*** defined as ***f*(u v)=|f(u)-f(v)|*** is bijective[4].

2. PRELIMINARIES

This section gives the basic ideas, which are very support for developing this paper.

DEFINITION 2.1

A graph G = {V(G), E(G)} is said to admit Even Graceful Labeling. If f: V (G)→ {0,1,2....2q-1} is injective and the induced function f*: E(G)→ {2,4,6...2q-2} defined as f*(u v)=|f(u)-f(v)| is bijective.
A graph which admits Even Graceful Labeling is called an Even Graceful Graph.

DEFINITION 2.2

The Jewel graph J_n is the graph with the vertex set $V(J_n)= \{v_x, v_y, u_x, u_y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n) = \{v_xu_x, v_xv_y, v_yu_x, v_yu_y, v_xu_i, u_yu_i : 1 \leq i \leq n\}$. The prime edge in a jewel graph is defined to be the edge joining the vertices v_y and u_x.

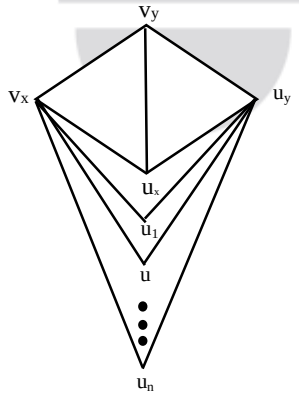

Fig 2.1 JEWEL GRAPH

DEFINITION 2.3

The Jewel Graph ***J*n*** without the prime edge is defined as the graph in which the prime edge, that is the edge joining the vertices v_y and u_x is removed.

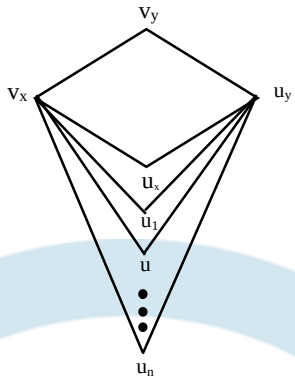

Fig 2.2 JEWEL GRAPH WITHOUT PRIME EDGE

DEFINITION 2.4

The Extended Jewel graph $EJ^*_{n,m}$ without the prime edge is the graph with the vertex set $V(EJ^*_{n,m}) = \{v_x, v_y, u_x, u_y, u_i, v_j : 1 \le i \le n, 1 \le j \le m\}$ and the edge set $E(EJ^*_{n,m}) = \{v_xu_x, v_xv_y, u_xu_y, v_yu_y, v_xu_y, u_yu_i, v_xv_j, u_yv_j : 1 \le i \le n, 1 \le j \le m\}$.

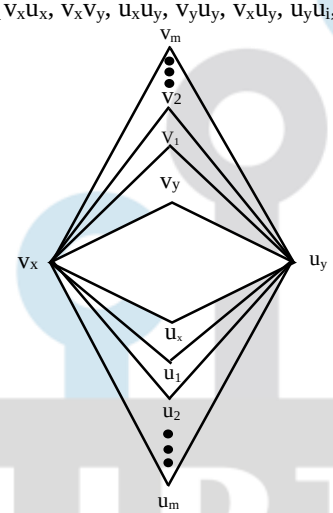

Fig 2.3 EXTENDED JEWEL GRAPH

3. THEOREMS ON JEWEL GRAPH AND EXTENDED JEWEL GRAPH

THEOREM 3.1

The Jewel graph J^*_n without the prime edge is Even Graceful.

PROOF

Let J^*_n be the Jewel graph without the prime edge where n denotes the number of jewels in the graph.

Let p and q be the number of vertices and edges respectively. The prime edge which joins the vertices v_y and u_x is removed.

The total number of vertices are given by $p = |V(J^*_n)| = n + 4$. The total number of edges are given by $q = |E(J^*_n)| = 2n + 4$.

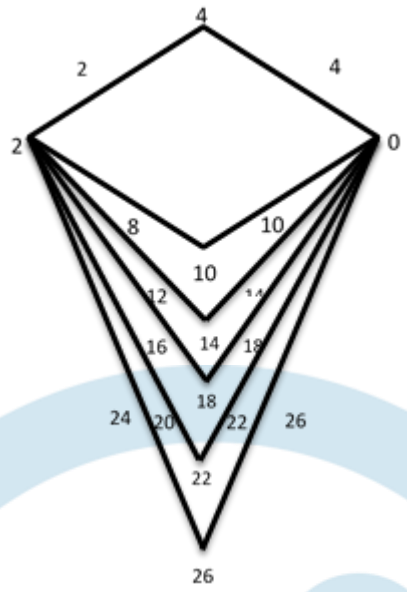

Fig 3.1 JEWEL GRAPH J^*_4 WITHOUT PRIME EDGE

Consider J^*_n with the vertex set $V(J^*_n) = \{v_x, v_y, u_x, u_y, u_i : 1 \le i \le n\}$ and the edge set $E(J^*_n) = \{v_xu_x, v_xv_y, v_yu_x, u_xu_y, v_yu_y, v_xu_i, u_yu_i : 1 \le i \le n\}$.

We define the vertex labeling $f : V(G) \rightarrow \{0, 1, 2, ..., 2q - 1\}$ as follows:

$f(u_x)=10$

$f(u_y)=0$

$f(v_x) = 2$

$f(v_y) = 4$

$f(ui) = 4i + 10, 1 \le i \le n$

We compute the edge labels as follows:

$|f(v_y) - f(v_x)| = 2$

$|f(v_y) - f(u_y)| = 4$

$|f(u_x) - f(v_x)| = 8$

$|f(u_x) - f(u_y)| = 4$

$|f(u_i) - f(u_y)| = 4i + 10, 1 \le i \le n,$

$|f(u_i) - f(v_x)| = 4i + 8, 1 \le i \le n.$

From the above computed edge labels we observe that the edge labels are distinct even numbers from the set ***{2,4,6….2q-2}.***

Hence the Jewel graph ***J*n*** without the prime edge is **Even Graceful.**

THEOREM 3.2

The Extended Jewel graph without prime edge is Even Graceful.

PROOF

Let $EJ^*_{n,m}$ be the extended jewel graph without the prime edge.

The total number of vertices are given by $|V(J^*_{n,m})| = m+n+4.$

The total number of edges are given by $|E(EJ^*_{n,m})| = 2(m+n+4)$

Consider $EJ^*_{n,m}$ with the vertex set $V(EJ^*_{n,m}) = \{v_x, v_y, u_x, u_y, u_i, v_j : 1 \le i \le n, 1 \le j \le m\}$ and
The edge set $E(EJ^*_{n,m}) = \{v_xu_x, v_xv_y, u_xu_y, v_yu_y, v_xu_i, u_yu_i, v_xv_j, u_yv_j : 1 \le i \le n, 1 \le j \le m\}$.

We define the vertex labeling $f : V(G) \rightarrow \{0, 1, 2, \ldots, 2q-1\}$ as follows:

$f(u_x)=10$

$f(u_y)=0$

$f(v_x)=2$

$f(v_y)=4$

$f(u_i) = 4i + 10, \quad 1 \le i \le n$

$f(v_j) = 4(n + j) + 10, 1 \le j \le m.$

We compute the edge labels $f : E(G) \rightarrow \{2,4,6...(2q-2)\}$ as follows:

$|f(v_y) - f(v_x)| = 2$

$|f(v_y) - f(u_y)| = 4$

$|f(u_x) - f(v_x)| = 8$

$|f(u_x) - f(u_y)| = 10$

$|f(u_i) - f(u_y)| = 4i + 10, 1 \le i \le n$

$|f(u_i) - f(v_x)| = 4i + 8, 1 \le i \le n$

$|f(v_j) - f(v_x)| = 4(n + j) + 8, 1 \le j \le m$

$|f(v_j) - f(u_y)| = 4(n + j) + 10, 1 \le j \le m$

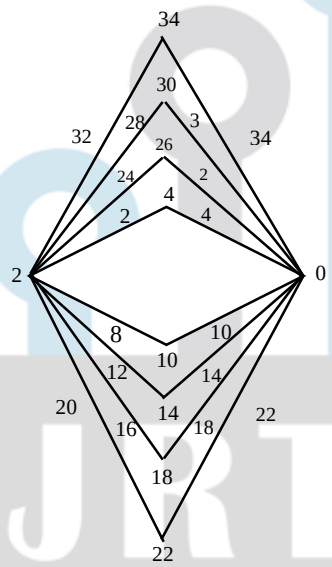

Fig 3.2 EXTENDED JEWEL GRAPH $J^*_{3,3}$ WITHOUT PRIME EDGE

From the above computed edge labels we observe that the edge labels are distinct even numbers from the set $\{2,4,6 \ldots ,(2q-2)\}$.

Hence the Extended Jewel graph $EJ^*_{n,m}$ without the prime edge is Even Graceful.

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