Performance Measures of a Repairable System with Multiple Units using Regenerative Point Technique and Semi-Markov Process

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Abstract- The primary objective of this paper is to evaluate the performance measures of a repairable system having two non-identical units (main and duplicate units) with the concept of change in environmental conditions. Initially, the main unit is operative whereas the duplicate unit is at cold standby. Single server visits the failed unit immediately for conducting repair. The expressions for some important reliability measures such as MTSF, Availability and Profit of the system model have been derived.

Index Terms- Performance measures, Repairable system, Environmental conditions.

I. INTRODUCTION

In view of the frequent and vital use in management and industrial sectors, the repairable systems of two or more identical units have been investigated stochastically in detail by the scholars including Gopalan and Naidu (1984), Goel and Sharma (1989) and Singh (1989) under strict control of environment conditions such as pollution, moisture, voltage, climate, temperature and other natural catastrophic. But in case of high cost of identical units, the non-identical unit (may be a substandard unit) might be kept as spare in cold standby not only to improve the reliability of the system but also to maintain performance of the system in emergency, as referred by Mokkadis et. al. (1989). Each unit is capable of performing the same kind of functions but their degree of reliability and desirability may differ from unit to unit. Deswal et al. (2013) discussed standby systems of non-identical units with different failure and repair policies. Also, sometimes it becomes very difficult to keep the environmental conditions under control which may fluctuate due changing climate and other natural catastrophic. While considering this fact in mind, Rajni (2013) and Barak (2019) and Kumar et. al. (2019), obtained reliability analysis of a cold standby system under different weather conditions. Further, the performance measures of cold standby repairable systems of non-identical units under different weather conditions have not been studied so far by the researchers in the field of reliability.

Hence, in the present Paper, a repairable system of two non-identical units – one is original (called main unit) and other is a substandard unit (called duplicate unit) has been analyzed stochastically in detail under two weather conditions – normal and abnormal. The environmental conditions when satisfied to the system correspond to normal weather; otherwise, it is supposed that the system is in abnormal weather. Initially, the system is operative with main unit and duplicate unit is kept a spare in cold standby. Both units have direct complete failure from normal mode. Each unit is capable of performing the same set of functions with different degree of reliability and desirability. There is a single server who visits the system immediately whenever needed to do repair of the failed unit in normal weather only. The operation and repair of the units are not allowed in abnormal weather as a precautionary measure to avoid excessive damage to the system. However, operation and repair of the units are as usual in normal weather. The units work as new after repair.

The distributions of failure time of the units and change of weather conditions follow negative exponential while that of repair times of the units are taken as arbitrary. All random variables are statistically independent. The switch devices and repairs are perfect. The expressions for various measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server and profit function in steady state are derived using semi-Markov process and regenerative point technique. The numerical results giving particular values to the parameters and various costs are obtained for mean time to system failure (MTSF), availability and profit to depict their tabular behavior. The application of the present work can be visualized in a software industry where application software is run through two different databases-one is initially operative and other is kept in cold standby.

II. NOTATIONS

E The set of regenerative states
MO/DO  
Main/Duplicate unit is good and operative  

MWO / DWO  
Main/Duplicate unit is good and waiting for operation in abnormal weather  

MCs/DCs  
Main/Duplicate unit is in cold standby mode  

MCs / DCs  
Main/Duplicate unit is in cold standby mode in abnormal weather  

$\lambda / \lambda$  
Constant failure rate of Main /Duplicate unit  

$\beta / \beta$  
Constant rate of change of weather from normal to abnormal/abnormal to normal weather  

MFur/DFur  
Main/duplicate unit failed and under repair  

MFUR/DFUR  
Main/duplicate unit failed and under repair continuously from previous state  

MFwr/DFwr  
Main/duplicate unit failed and waiting for repair  

MFWR/DFWR  
Main/duplicate unit failed and waiting for repair continuously from previous state  

MFWR / DFwr  
Main/Duplicate unit failed and waiting for repair due to abnormal weather  

MFWR / DFWR  
Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather  

g(t)/G(t)  
pdf/cdf of repair time of Main unit  

$g_1(t)/G_1(t)$  
pdf/cdf of repair time of Duplicate unit  

$q_{ij}(t)/Q_{ij}(t)$  
pdf/cdf of passage time from regenerative state $i$ to regenerative state $j$ or to a failed state $j$ without visiting any other regenerative state in $(0,t]$  

$q_{ij,k,r}(t)/Q_{ij,k,r}(t)$  
pdf/cdf of direct transition time from regenerative state $i$ to a regenerative state $j$ or to a failed state $j$ visiting state $k,r$ once in $(0,t]$  

$m_{ij}$  
The conditional mean sojourn time in regenerative state $Si$ when system is to make transition in to regenerative state $Sj$. Mathematically, it can be written as  

$$m_{ij} = E(T_{ij}) = \int_0^\infty t d[Q_{ij}(t)] = -Q_{ij}^{*}(0)^*,$$  

where $T_{ij}$ is the transition time from state $Si$ to $Sj$; $Si$, $Sj \in E$.  

$\mu_{ij}$  
The mean Sojourn time in state $Si$ this is given by  

$$\mu_{ij} = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij},$$  

where $T_i$ is the sojourn time in state $Si$.  

$\mathbb{S}/\mathbb{G}/\mathbb{G}^n$  
Symbol for Laplace Stieltjes convolution/Laplace convolution/Laplace convolution $n$ times  

** / *  
Symbol for Laplace Steiltjes Transform (L.S.T.)/ Laplace transform (L.T.)  

$'$(desh)  
Used to represent alternative result  

The following are the possible transition states of the system  

$S_0 = (MO, DCs), S_1 = (MFur, DO), S_2 = (MWO, DCs), S_3 = (MFwr, DWO), S_4 = (MFUR, DFwr), S_5 = (MCs, DO), S_6 = (MO, DFur), S_7 = (MFwr, DFWR), S_8 = (MFur, DFWR), S_9 = (MCs, DWO), S_{10} = (MWO, DFwr),$.  

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\[ S_{11} = (MFwr, DFUR), \quad S_{12} = (MFWR, DFWR) \]

The states \( S_0, S_1, S_2, S_3, S_5, S_6, S_9 \) and \( S_{10} \) are regenerative while the states \( S_4, S_7, S_8, S_{11}, S_{12} \) and \( S_{13} \) are non-regenerative as shown in below figure

**Transition Diagram of the Proposed Model**

III. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The differential transition probabilities are:

\[
\begin{align*}
dQ_{01}(t) &= \lambda e^{-(\lambda+\beta)t} dt, \\
dQ_{02}(t) &= \beta e^{-(\lambda+\beta)t} dt, \\
dQ_{13}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{14}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{15}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{16}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{20}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{20}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{20}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{20}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{20}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{20}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{20}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt, \\
dQ_{20}(t) &= \lambda e^{-(\lambda+\beta)t} G(t) dt, \\
dQ_{20}(t) &= \beta e^{-\beta t} e^{-(\lambda+\beta)t} dt.
\end{align*}
\]
Simple probabilistic considerations yield the following expressions for the non-zero elements

\[ p_{ij} = \frac{\lambda_j}{\beta + \lambda_j}, \quad p_{02} = \frac{\beta}{\beta + \lambda_j}, \quad p_{13} = \frac{\beta}{\beta + \lambda_j} (1 - g^*(\beta + \lambda_j)), \quad p_{14} = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)), p_{15} = g^*(\beta + \lambda_1), \]

\[ p_{20} = 1, \quad p_{31} = 1, \quad p_{46} = g^*(\beta), p_{47} = 1 - g^*(\beta), p_{56} = \frac{\lambda_1}{\beta + \lambda_1}, \quad p_{58} = \frac{\beta}{\beta + \lambda_1} p_{56} = g^*(\beta + \lambda_1), \]

\[ p_{610} = \frac{\beta}{\beta + \lambda} (1 - g^*(\beta + \lambda)), p_{611} = \frac{\lambda}{\beta + \lambda} (1 - g^*(\beta + \lambda)), p_{78} = 1, \quad p_{56} = g^*(\beta), \]

\[ p_{87} = 1 - g^*(\beta), \quad p_{56} \equiv 1, p_{10,6} = 1, p_{11,1} = g^*(\beta), p_{11,12} = g^*(\beta), p_{12,13} = 1, p_{13,1} = g^*(\beta), \]

\[ p_{13,1} = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)) g^*(\beta), p_{16.4,7,8} = \frac{\lambda_1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)) (1 - g^*(\beta)), \]

\[ p_{611} = \frac{\lambda}{\beta + \lambda} (1 - g^*(\beta + \lambda)) g^*(\beta), p_{61,11,12,13} = \frac{\lambda}{\beta + \lambda} (1 - g^*(\beta + \lambda)) (1 - g^*(\beta)) \]

\[ ... (2) \]

It can be easily verified that

\[ p_{01} + p_{02} = p_{13} + p_{16.4,7,8} + p_{15} = p_{20} + p_{31} + p_{46} + p_{47} = p_{56} = 1, \]

\[ p_{610} + p_{611} + p_{61,11,12,13} = p_{78} = 1, \quad p_{11,12} = p_{12,13} = 1, \quad p_{13,1} + p_{13,12} = 1 \]

\[ ... (3) \]

The mean sojourn times (\(\mu_i\)) in the state \(S_i\) are

\[ \mu_0 = m_{00} + m_{02} = \frac{1}{\beta + \lambda_j}, \quad \mu_1 = m_{13} + m_{14} + m_{15} = \frac{1}{\beta + \lambda_1} (1 - g^*(\beta + \lambda_1)), \quad \mu_2 = m_{20} = \frac{1}{\beta_1}, \]

\[ \mu_3 = m_{31} = \frac{1}{\beta_1}, \quad \mu_4 = m_{46} + m_{47} = \frac{1}{\beta} (1 - g^*(\beta)), \quad \mu_5 = m_{56} + m_{59} = \frac{1}{\beta + \lambda_1}, \]

\[ \mu_6 = m_{60} + m_{6.10} + m_{6.11} = \frac{1}{\beta + \lambda} (1 - g^*(\beta + \lambda_1)), \]

\[ \mu_7 = m_{78} = \frac{1}{\beta_1}, \quad \mu_8 = m_{86} + m_{87} = \frac{1}{\beta} (1 - g^*(\beta)), \quad \mu_9 = m_{98} = \frac{1}{\beta_1}, \quad \mu_{10} = m_{10.6} = \frac{1}{\beta_1}, \]

\[ \mu_{11} = m_{11,1} + m_{11,12} = \frac{1}{\beta} (1 - g^*(\beta)), \quad \mu_{12} = m_{12,1} = \frac{1}{\beta_1}, \quad \mu_{13} = m_{13,1} + m_{13,12} = \frac{1}{\beta} (1 - g^*(\beta)), \]

\[ \mu'_{1} = m_{13} + m_{15} + m_{16.4,7,8} = \frac{1 - g^*(\beta + \lambda_1)) (\beta \beta \beta_1 g^*(\beta) + \lambda_1 (\beta + \beta_1) (1 - g^*(\beta))}{\beta \beta_1 (\beta + \lambda_1) g^*(\beta)}, \]

\[ \mu'_{6} = m_{60} + m_{611} + m_{61,11,12,13} + m_{6.10} = \frac{(1 - g^*(\beta + \lambda_1)) (\beta \beta_1 g^*(\beta) + \lambda_1 (\beta + \beta_1) (1 - g^*(\beta))}{\beta \beta_1 (\beta + \lambda_1) g^*(\beta)} \]

\[ ... (4) \]

IV. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let \(Q_i(t)\) be the cdf of first passage time from regenerative state \(S_i\) to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for \(Q_i(t)\):

\[ Q_0(t) = Q_{01}(t) \mathbb{S} Q_1(t) + Q_{02}(t) \mathbb{S} Q_2(t) \]

\[ Q_1(t) = Q_{13}(t) \mathbb{S} Q_3(t) + Q_{14}(t) \mathbb{S} Q_5(t) + Q_{14}(t) \]

\[ Q_2(t) = Q_{20}(t) \mathbb{S} Q_0(t) \]

\[ Q_3(t) = Q_{31}(t) \mathbb{S} Q_1(t) \]

\[ Q_5(t) = Q_{56}(t) \mathbb{S} Q_6(t) + Q_{59}(t) \mathbb{S} Q_9(t) \]
\[ \emptyset_6(t) = Q_{60}(t) \emptyset_0(t) + Q_{6,10}(t) \emptyset_10(t) + Q_{6,11}(t) \]
\[ \emptyset_9(t) = Q_{95}(t) \emptyset_5(t) \]
\[ \emptyset_{10}(t) = Q_{10,6}(t) \emptyset_6(t) \]
\[ \ldots (5) \]

Taking L.S.T. of above relations (5) and solving for \( \emptyset_0^{**}(s) \), we get
\[ \emptyset_0^{**}(s) = \frac{ Q_{01}^{**}(s) Q_{15}^{**}(s)(1-Q_{6,10}^{**}(s) Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s) Q_{95}^{**}(s)) + Q_{01}^{**}(s) Q_{56}^{**}(s) Q_{61}^{**}(s)}{(1-Q_{02}^{**}(s) Q_{20}^{**}(s))(1-Q_{13}^{**}(s) Q_{31}^{**}(s))(1-Q_{6,10}^{**}(s) Q_{10,6}^{**}(s))(1-Q_{59}^{**}(s) Q_{95}^{**}(s))} \]
\[ -Q_{01}^{**}(s) Q_{15}^{**}(s) Q_{56}^{**}(s) Q_{60}^{**}(s) \]
\[ \ldots (6) \]

we have
\[ R^*(s) = \frac{1 - \emptyset_0^{**}(s)}{s} \]
\[ \ldots (7) \]

The reliability of the system can be obtained by taking inverse Laplace transform of (7).

The mean time to system failure (MTSF) is given by
\[ MTSF = \lim_{s \to 0} \frac{1 - \emptyset_0^{**}(s)}{s} = \frac{N_i}{D_i} \]
\[ \ldots (8) \]

where
\[ N_i = p_{56}(1-p_{6,10})(1-p_{13})(\mu_0+p_0 \mu_2)+p_{56}(1-p_{6,10})(\mu_3+p_0 \mu_3)+p_{56}(1-p_{6,10})(\mu_5+p_5 \mu_9)+p_{56}(1-p_{6,10}(\mu_6+p_6 \mu_{10})) \]
\[ D_i = p_{56}(1-p_{6,10})(1-p_{13})(1-p_{6,10})p_{60} \]
\[ \ldots (9) \]

V. STEADY STATE AVAILABILITY

Let \( A_i(t) \) be the probability that the system is in up-state at instant \( t \) given that the system entered regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( A_i(t) \) are given as:
\[ A_0(t) = M_0(t) + q_{01}(t) \bowtie A_1(t) + q_{02}(t) \bowtie A_2(t) \]
\[ A_1(t) = M_1(t) + q_{13}(t) \bowtie A_3(t) + q_{11}(t) \bowtie A_0(t) + q_{16.4}(t)+q_{16.4}(7,8)N(t) \bowtie A_0(t) \]
\[ A_2(t) = q_{20}(t) \bowtie A_0(t) \]
\[ A_3(t) = q_{31}(t) \bowtie A_1(t) \]
\[ A_4(t) = M_4(t) + q_{45}(t) \bowtie A_5(t) + q_{49}(t) \bowtie A_0(t) \]
\[ A_5(t) = M_5(t) + q_{56}(t) \bowtie A_6(t) + q_{59}(t) \bowtie A_0(t) \]
\[ A_6(t) = M_6(t) + q_{60}(t) \bowtie A_0(t) + q_{6,10}(t) A_1(t) + q_{6,10}(t) A_10(t) \]
\[ A_7(t) = q_{6,10}(t) \bowtie A_0(t) \]
\[ \ldots (10) \]

where \( M_i(t) \) is the probability that the system is up initially in state \( S_i \in E \) is up at time \( t \) without visiting to any other regenerative state, we have
\[ M_0(t) = e^{-(\beta+\lambda)t} , M_1(t) = e^{-(\beta+\lambda)t} G_1(t) , M_5(t) = e^{-(\beta+\lambda)t} , M_6(t) = e^{-(\beta+\lambda)t} G_1(t) \]
\[ \ldots (11) \]

Taking L.T. of above relations (10) and (11) and solving for \( A_0^{**}(s) \), we have
(M_0(s)(1-q_{13}(s)q_{31}(s)) + q_{01}(s)M_1(s)(1-q_{6,10}(s)q_{10,6}(s))(1-q_{59}(s)q_{59}(s))
-(q_{61,11}(s) + q^{*}_{61,11}(s,12,13)^{p}(s))M_2(s)(q_{15}(s)q_{56}(s) + (q_{16,4}(s) + q^{*}_{16,4}(7,8)^{p}(s)))
(1-q_{59}(s)q_{59}(s)) + q_{01}(s)q_{15}(s)(1-q_{6,10}(s)q_{10,6}(s))M_2(s) +
\frac{q_{56}(s)M_6(s)(q_{16,4}(s) + q^{*}_{16,4}(7,8)^{p}(s))q_{51}(s)(1-q_{59}(s)q_{59}(s))M_6(s)}{(1-q_{6,10}(s)q_{10,6}(s))(1-q_{59}(s)q_{59}(s))}(1-q_{59}(s)q_{59}(s))}
\frac{A_0(s)}{D_2} = \frac{\beta \lambda}{(\beta \lambda + \mu)^{2}}
\lim_{t \to \infty} A_0(s) = \frac{N_2}{D_2}
\ldots(13)
N_2 = \frac{\beta \lambda}{(\beta \lambda + \mu)^{2}}
D_2 = \frac{\beta \lambda}{(\beta \lambda + \mu)^{2}}
\ldots(14)

VI. BUSY PERIOD ANALYSIS OF THE SERVER

Let B(t) be the probability that the server is busy in repairing the unit at an instant ‘t’ given that the system entered regenerative state S, at t=0. The recursive relations for B(t) are as follows:
B_0(t) = q_{01}(t) \cap B_1(t) + q_{02}(t) \cap B_2(t)
B_1(t) = W_1(t) + q_{13}(t) \cap B_1(t) + q_{15}(t) \cap B_2(t) + (q_{16,4}(t) + q^{*}_{16,4}(7,8)^{p}(t)) \cap B_1(t)
B_2(t) = q_{20}(t) \cap B_0(t)
B_1(t) = q_{13}(t) \cap B_0(t)
B_2(t) = q_{56}(t) \cap B_0(t) + q_{59}(t) \cap B_0(t)
B_3(t) = W_3(t) + q_{60}(t) \cap B_3(t) + (q_{61,11}(t) + q^{*}_{61,11}(s,12,13)^{p}(t)) \cap B_1(t) + q_{6,10}(t) \cap B_1(t)
B_2(t) = q_{59}(t) \cap B_3(t)
B_1(t) = q_{10,6}(t) \cap B_0(t)
\ldots(15)

where W(t) be the probability that the server is busy in state S, due to failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative states so,
W(t) = e^{-\beta \lambda t} \frac{q_{01}(t)}{G(t)} + (\lambda \mu e^{-(\beta \lambda + \mu)T} \cap 1) \frac{G(t)}{G(t)} \cdot W_0(t) = e^{-\beta \lambda t} \frac{q_{01}(t)}{G(t)} + (\lambda \mu e^{-(\beta \lambda + \mu)T} \cap 1) \frac{G(t)}{G(t)}
\ldots(16)

Taking L.T. of above relations (15) and (16) and solving for B_0^*(s). We obtain
B_0^*(s) = \frac{q_{01}(s)W_0^*(s)(q_{15}(s)q_{56}(s) + (q_{16,4}(s) + q^{*}_{16,4}(7,8)^{p}(s))(1-q_{59}(s)q_{59}(s)))}{(1-q_{6,10}(s)q_{10,6}(s))(1-q_{59}(s)q_{59}(s))}(1-q_{59}(s)q_{59}(s))}
\frac{A_0(s)}{D_2} = \frac{N_3}{D_2}
\lim_{s \to 0} sB_0^*(s) = \frac{N_3}{D_2}

The time for which server is busy due to repair is given by
B_0^*(t) = \frac{W_0^*(s)}{D_2}
\ldots(17)
where

\[ N_3 = p_0 p_5 q (W_1^* (0)(1-p_{6.10}) + W_6^* (0)(1-p_{13})) \] and \( D_2 \) is already defined

...(19)

**VII. EXPECTED NUMBER OF VISITS BY THE SERVER**

Let \( N_i(t) \) be the expected number of visits by the server in \((0,t] \) given that the system entered the regenerative state \( S_i \) at \( t=0 \). The recursive relations for \( N_i(t) \) are given as

\[ N_0(t) = Q_{01}^0(t) \cdot (1+N_1(t)) + Q_{02}^0(t) \cdot N_2(t) \]

\[ N_1(t) = Q_{13}^1(t) \cdot N_3(t) + Q_{15}^1(t) \cdot N_5(t) + Q_{16.4}^1(t) \cdot N_6(t) + Q_{16.4,7.8}^1(t) \cdot (1+N_6(t)) \]

\[ N_2(t) = Q_{20}^2(t) \cdot N_0(t) \]

\[ N_3(t) = Q_{31}^3(t) \cdot N_4(t) \]

\[ N_5(t) = Q_{56}^5(t) \cdot N_2(t) \]

\[ N_6(t) = Q_{60}^6(t) \cdot N_0(t) + Q_{61.11}^6(t) \cdot N_9(t) + Q_{61.11,12.13}^6(t) \cdot (1+N_6(t)) + Q_{6.10}^6(t) \cdot N_{10}(t) \]

\[ N_9(t) = Q_{95}^9(t) \cdot N_5(t) \]

\[ N_{10}(t) = Q_{10.6}^{10}(t) \cdot (1+N_6(t)) \]

...(20)

Taking L.S.T. of relations (20) and solving for \( N^*_0(s) \), we get

\[ N^*_0(s) = \frac{Q_{01}^0(s)(1+Q_{16.4,7.8}^1(s))(1-Q_{16.4,11,12,13}^1(s))(1-Q_{16.4}^1(s)Q_{95}^1(s))}{(1-Q_{16.4}^1(s)Q_{95}^1(s))(1-Q_{15}^1(s)Q_{65}^1(s)Q_{56}^1(s)+Q_{16.4}^1(s)Q_{95}^1(s))} \]

...(21)

The expected numbers of visits per unit time by the server are given by

\[ N_0(\infty) = \lim_{s \to 0} s N^*_0(s) = \frac{N_4}{D_2} \]

...(22)

where

\[ N_3 = p_0 p_5 q (W_1^* (0)(1-p_{6.10}) + W_6^* (0)(1-p_{13})) \] and \( D_2 \) is already specified.

...(23)

**VIII. PROFIT ANALYSIS**

The profit incurred to the system model in steady state can be obtained as

\[ P = K_0 A_0 - K_1 B_0 - K_2 N_0 \]

where

\[ K_0 = \text{Revenue per unit up-time of the system} \]

\[ K_1 = \text{Cost per unit for which server is busy} \]

\[ K_2 = \text{Cost per unit visit by the server} \]

and \( A_0, B_0, N_0 \) are already defined.

**IX. PARTICULAR CASE**

Suppose \( g(t) = e^{-\alpha t}, g_1(t) = \alpha_1 e^{-\alpha_1 t} \)
By using the non-zero elements $p_{ij}$, we can obtain the following results:

\[
p_{01} = \frac{\lambda}{\beta + \lambda}, \quad p_{02} = \frac{\beta}{\beta + \lambda}, \quad p_{13} = \frac{\beta}{\alpha + \beta + \lambda_1}, \quad p_{14} = \frac{\lambda_3}{\alpha + \beta + \lambda_4}, \quad p_{15} = \frac{\alpha}{\alpha + \beta + \lambda_5}, \quad p_{20} = 1, \quad p_{31} = 1,
\]

\[
p_{46} = \frac{\alpha}{\alpha + \beta}, \quad p_{47} = \frac{\beta}{\alpha + \beta}, \quad p_{56} = \frac{\lambda}{\alpha + \beta + \lambda_1}, \quad p_{59} = \frac{\beta}{\alpha + \beta + \lambda_4}, \quad p_{60} = \frac{\lambda_4}{\alpha + \beta + \lambda_5}, \quad p_{610} = \frac{\alpha}{\alpha + \beta + \lambda}, \quad p_{611} = \frac{\beta}{\alpha + \beta},
\]

\[
p_{611,12} = \frac{\beta}{\alpha_1 + \beta + \lambda}, \quad p_{12,13} = 1, \quad p_{13,1} = \frac{\alpha_1}{\alpha_1 + \beta + \lambda_1}, \quad p_{13,12} = \frac{\beta}{\alpha_1 + \beta + \lambda_1}, \quad p_{16,4} = \frac{\lambda_4}{\alpha_1 + \beta + \lambda_5}, \quad p_{16,7} = \frac{\beta}{\alpha + \beta + \lambda_5},
\]

\[
p_{611,12,13} = \frac{\beta}{\alpha_1 + \beta + \lambda_5}, \quad \mu_3 = \frac{1}{\beta_1}, \quad \mu_5 = \frac{1}{\beta_1}, \quad \mu_6 = \frac{1}{\alpha + \beta + \lambda_1}, \quad \mu_7 = \frac{1}{\alpha + \beta + \lambda_1}, \quad \mu_9 = \frac{1}{\alpha + \beta + \lambda_1}, \quad \mu_12 = \frac{1}{\alpha_1 + \beta + \lambda_1}, \quad \mu_13 = \frac{1}{\alpha_1 + \beta + \lambda_1},
\]

\[
W^*(0) = \frac{(\alpha + \lambda_1)}{\alpha_1 + \beta + \lambda_1}, \quad W^*(0) = \frac{(\alpha + \lambda_1)}{\alpha_1 + \beta + \lambda_1},
\]

\[
MTSF (T_0) = \frac{N_3}{D_1}, \quad \text{Steady state availability} (A_0) = \frac{N_2}{D_2},
\]

Busy period of the server ($B_0$) = \frac{N_4}{D_2},

Expected number of visits by the server ($N_0$) = \frac{N_4}{D_2},

where

\[
N_1 = (\beta + \beta_1)(\lambda_1(\alpha_1 + \lambda)(\alpha + \lambda + \lambda_1) + \alpha(\alpha_1 + \lambda + \lambda_1))
\]

\[
D_1 = \lambda_1(\beta_1(\alpha + \lambda_1)(\alpha_1 + \lambda_1))
\]

\[
N_2 = \alpha(\alpha_1 + \beta_1)(\alpha_1(\alpha_1 + \lambda_1))
\]

\[
D_2 = \alpha(\alpha_1 + \beta_1)(\alpha_1(\alpha_1 + \lambda_1)(\alpha_1 + \lambda_1)) + \alpha(\alpha_1 + \lambda_1)(\alpha_1(\alpha_1 + \lambda_1)(\alpha_1 + \lambda_1))
\]

\[
N_3 = \alpha(\alpha_1 + \beta_1)(\alpha_1(\alpha_1 + \lambda_1)(\alpha_1 + \lambda_1))
\]

\[
N_4 = \frac{\alpha(\alpha_1 + \beta_1)(\alpha_1 + \beta_1)(\alpha_1 + \beta)(\alpha_1 + \lambda_1) + \alpha(\alpha_1 + \lambda_1)(\alpha_1(\alpha_1 + \lambda_1)(\alpha_1 + \lambda_1))}{(\alpha + \beta)(\alpha_1 + \beta_1)}
\]

\[
X. TABLES
\]

<table>
<thead>
<tr>
<th>Normal Weather Rate($\beta_1$)</th>
<th>$\alpha=2, \alpha_1=2.5$, $\beta=0.01$, $\lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5$</th>
<th>$\alpha_1=2$, $\beta=0.05$</th>
<th>$\lambda=0.3$</th>
<th>$\lambda_1=0.4$</th>
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<tbody>
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<td>11.01942286</td>
<td>9.727451</td>
<td>10.41944</td>
<td>11.38393</td>
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<tr>
<td>1.3</td>
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<td>9.721267</td>
<td>10.41282</td>
<td>11.3489</td>
<td>16.97166</td>
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<tr>
<td>1.5</td>
<td>11.00142857</td>
<td>9.711373</td>
<td>10.40222</td>
<td>11.29286</td>
<td>16.95439</td>
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Table 2: Availability vs. Normal Weather Rate ($\beta_1$)

<table>
<thead>
<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5, \alpha_1=2$</th>
<th>$\beta=0.05, \lambda=0.3, \lambda_1=0.4$</th>
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<td>0.952288701, 0.93391, 0.944042, 0.950654, 0.969052</td>
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<tr>
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<td>0.952322813, 0.933956, 0.944081, 0.950824, 0.969075</td>
<td>0.967482</td>
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</tr>
<tr>
<td>1.3</td>
<td>0.952351679, 0.933995, 0.944115, 0.950968, 0.969094</td>
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<tr>
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<td>0.967519</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>0.952397867, 0.934058, 0.944169, 0.951198, 0.969124</td>
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<td></td>
</tr>
<tr>
<td>1.6</td>
<td>0.952416632, 0.934083, 0.94419, 0.951292, 0.969137</td>
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</tr>
<tr>
<td>1.7</td>
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</tr>
<tr>
<td>1.8</td>
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<tr>
<td>1.9</td>
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<tr>
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Table 3: Profit vs. Normal weather rate ($\beta_1$)

<table>
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<tr>
<th>Normal Weather Rate ($\beta_1$)</th>
<th>$K_0=5000, K_1=350, K_2=300$</th>
<th>$\alpha=2, \alpha_1=2.5, \beta=0.01, \lambda=0.5, \lambda_1=0.6$</th>
<th>$\alpha=1.5, \alpha_1=2$</th>
<th>$\beta=0.05, \lambda=0.3, \lambda_1=0.4$</th>
</tr>
</thead>
<tbody>
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<td>1.1</td>
<td>4577.52164, 4476.754344, 4531.372, 4564.752, 4704.575</td>
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<td></td>
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<tr>
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<tr>
<td>1.3</td>
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<tr>
<td>1.4</td>
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<tr>
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<td>4693.929</td>
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<td></td>
</tr>
</tbody>
</table>

XI. CONCLUSION
To make the study more concrete, the numerical results giving particular values to the various parameters and cost are obtained to depict the behavior of Mean Time to System Failure (MTSF), availability and profit functions as shown respectively in Table 1, 2 and 3. It is revealed that MTSF decreases with the increase in normal weather rate ($\beta_1$) and failure rates ($\lambda, \lambda_1$) of the units. And, it increases with increase of abnormal weather rate ($\beta$) and repair rates ($\alpha, \alpha_1$) of the units. The results show that availability and profit of the system model keep on increasing as normal weather rate ($\beta_1$) and repair rates ($\alpha, \alpha_1$) increase while their values decline with the increase of abnormal weather rate ($\beta$) and failure rates ($\lambda, \lambda_1$) of the units. Thus, on the basis of the results obtained for a particular case, it is interpreted that a system of non-identical units which is not allowed to operate in abnormal weather conditions can be made more available and profitable to use either by providing normal weather for operation or by providing better repair facilities like calling server of high repair rates (may be called as expert server).

REFERENCES: