Intuitionistic Fuzzy gsr Cokernal Compact Spaces

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Abstract – In this paper, we introduced and studied the concepts of intuitionistic fuzzy gsr- cokernal compact spaces, intuitionistic fuzzy gsr C- compact continuous function and intuitionistic fuzzy gsr R-compact spaces.

Index Terms- Intuitionistic fuzzy-gsr-C-compact set, Intuitionistic fuzzy-gsr-C-cocompact set, Intuitionistic fuzzy gsr cokernal compact space.

1. INTRODUCTION

The concept of generalized closed sets in general topology was brought into light by Levine [5]. Later its properties such as continuity, connectedness and compactness were studied. All these properties of generalized closed sets were extended to intuitionistic fuzzy set by Atanassov [2]. Generalized closed sets were further developed as gsr closed sets in soft topological spaces by Mohana et.al [6]. The same gsr closed sets in intuitionistic fuzzy topology was discussed by Anitha and Mohana and [1]. In this paper, IFgsr cokernal compact spaces evolved as a result of the idea proposed by Roja et.al [8] who introduced cokernal compact spaces in intuitionistic topology and the notion of Igsr cokernal compact spaces was introduced and investigated by Mohana and Stepby Stephen [10].

2. PRELIMINARIES

Definition 2.1. [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS’s in X containing Φ, X and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS in short). Any intuitionistic set in τ is known as intuitionistic open set (IOS) in X and the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 2.2. [2] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu_A(x) : X \rightarrow [0,1]$ and $\nu_A(x) : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.3. [2] $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A' = \{< x, \nu_A(x), \mu_A(x) > / x \in X\}$
- $A \cap B = \{< x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) > / x \in X\}$
- $A \cup B = \{< x, \nu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) > / x \in X\}$

Definition 2.4. [2] An intuitionistic fuzzy topology (IFT in short) on X is a family $\tau$ of IFSs in X satisfying the following axioms.

- $0, \tau \in \tau$
- $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
- $\cup G_\tau \in \tau$ for any family $\{G_i : i \in I\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement $A^c$ of an IFOS A in an IFTS(X,τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.5. [1] An IFS A in an IFTS (X,τ) is said to be an Intuitionistic fuzzy gsr-closed sets (IFGSRCS in short) if $\text{scl}(A) \subseteq U$ and U is an IFROS in (X,τ).

Definition 2.6. [10] Let $(X,\tau)$ be an intuitionistic topological space. Then $A = \{X, A_1, A_2 > \in \tau$ is said to be intuitionistic $\text{gsr-C}$-compact (IgsrC-compact) set if every $A \subseteq \bigcup_{i \in I} A_i^c$ where $A_i^c$ is Igsr-closed set in $(X,\tau)$. The complement of an intuitionistic-gsr-C-compact set is an intuitionistic-gsr-C-cocompact (IgsrC-cocompact) set.

Definition 2.7. [10] Let $(X, \tau)$ be an intuitionistic topological space and $A = \{X, A_1, A_2 > \in \tau$ be an intuitionistic set in $(X, \tau)$. Then the IgsrC-compact kernel of A and IgsrC-compact cokernal of A are denoted and defined by
\( IgsrK^- (A) = \bigcup \{ K = < X, K_1, K_2 > : K \text{ is an } Igsr\text{-compact set in } (X, \tau) \text{ and } K \subseteq A \} \)

\( IgsrCK^- (A) = \cap \{ K = < X, K_1, K_2 > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } A \subseteq K \} \)

**Remark 2.8.**[10] Let \((X, \tau)\) be an intuitionistic topological space and \(A = < X, A_1, A_2 > \) be an intuitionistic set of \(X\). Then the following conditions hold:

i) \( IgsrCK^- (A) = A \Leftrightarrow A \text{ is an } Igsr\text{-cocompact set.} \)

ii) \( IgsrK^- (A) = A \Leftrightarrow A \text{ is an } Igsr\text{-compact set.} \)

**Definition 2.9.**[10] An intuitionistic topological space \((X, \tau)\) is said to be an \(Igsr\) cokernel compact space if the \(Igsr\)-cocompact cokernel of every \(Igsr\)-compact set. That is, \( IgsrK^-c(x) = \bigcup A^c. \)

### 3. INTUITIONISTIC FUZZY GSR COKERNEL COMPACT SPACES

**Definition 3.1:** Let \((X, \tau)\) be an intuitionistic fuzzy topological space. Then \(A = < x, \mu(x), v(x) > \in \tau\) is said to be intuitionistic fuzzy gsr-C-compact (IFgsrC-compact) set if every \(A \subseteq \bigcup A_i\) where \(A_i\) is IFgsr-compact set in \((X, \tau)\).

The complement of an intuitionistic fuzzy gsr-C-compact is an intuitionistic fuzzy gsr-C-compact (IFgsrC-compact) set.

**Definition 3.2:** Let \((X, \tau)\) be an intuitionistic fuzzy topological space and \(A = < x, \mu(x), v(x) > \in \tau\) is said to be intuitionistic fuzzy gsr-C-compact (IFgsrC-compact) set if every IFgsr-compact kernel of \(A\) and IFgsr-compact cokernel of \(A\) are defined by\(\)

\[ IFgsrK^- (A) = \bigcup \{ K = < x, \mu_k(x), v_k(x) > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } K \subseteq A \} \]

\[ IFgsrCK^- (A) = \cap \{ K = < x, \mu_k(x), v_k(x) > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } A \subseteq K \} \]

**Remark 3.3:** Let \((X, \tau)\) be an intuitionistic fuzzy topological space and \(A = < x, \mu(x), v(x) > \in \tau\) is said to be intuitionistic fuzzy gsr-C-compact set. Then

- \( IFgsrK^- (A) = A \) if and only if \( A \) is an IFgsr-C-compact set.
- \( IFgsrK^- (A) = A \) if and only if \( A \) is an IFgsr-C-compact set.

**Definition 3.4:** An intuitionistic fuzzy topological space \((X, \tau)\) is said to be an IFgsr cokernel compact space if the IFgsr-compact cokernel of every IFgsr-compact set. That is, \( IFgsrK^-c(x) = \bigcup A^c. \)

**Example 3.5:** Let \(X = \{ a, b \}\), then the intuitionistic fuzzy set \(A = < X, (0.5,0.4),(0,2,0.3)>, B = < X, (0,4,0.5),(0.3,0.4)>, C = < X, (0,6,0.5),(0,1,0.4)>, D = < X, (0,3,0,3),(0,6,0.5)>, E = < X, (0,2,0.4),(0,4,0.5)>, F = < X, (0,1,0,3),(0,6,0,5)>. \)

Then the family \(\tau = \{ \emptyset, X, A, B, C, \} \) is an intuitionistic fuzzy topology on \(X\).

\[ IFgsrC-compact set = \{ < X, (0,5,0,4),(0,2,0,3)>, < X, (0,4,0,5),(0,3,0,4)>, < X, (0,6,0,5),(0,1,0,4)>, < X, (0,3,0,3),(0,6,0,5)>, < X, (0,2,0,4),(0,4,0,5)>, < X, (0,1,0,3),(0,6,0,5) > \} \]

\[ IFgsrC-cocompact set = \{ < X, (0,2,0,3),(0,5,0,4)>, < X, (0,3,0,4),(0,4,0,5)>, < X, (0,1,0,4),(0,6,0,5) >, < X, (0,6,0,5),(0,3,0,3) >, < X, (0,4,0,5),(0,2,0,4) >, < X, (0,6,0,5),(0,1,0,3) > \} \]

Then \( IFgsrK^-c(x) = \bigcup K \). Therefore, \((X, \tau)\) is IFgsr cokernel compact space.

**Proposition 3.6:** Let \((X, \tau)\) be any intuitionistic fuzzy topological space. Let \(A = < x, \mu(x), v(x) > \in \tau\) be an IFgsrC-compact set in \(X\). Then the following conditions hold:

i) \( IFgsrKR^- (A) = IFgsrK^r (\bar{A}) \).

ii) \( IFgsrKR^- (A) = IFgsrK^r (\bar{A}) \).

**Proof:** i) \( IFgsrKR^- (A) = \bigcap \{ K = < x, \mu_k(x), v_k(x) > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } K \subseteq A \} \).

Taking complements on both sides,

\[ IFgsrKR^- (A) = \bigcup \{ K = < x, \mu_k(x), v_k(x) > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } A \subseteq K \} \]

ii) \( IFgsrKR^- (A) = IFgsrK^r (\bar{A}) \).

Taking complements on both sides,

\[ IFgsrK^r (\bar{A}) \cap \{ K = < x, \mu_k(x), v_k(x) > : K \text{ is an } Igsr\text{-cocompact set in } (X, \tau) \text{ and } A \subseteq K \} \]
= \text{IFgsrCK}_c(\overline{A}).

**Proposition 3.7:** Let \((X, \tau)\) be an intuitionistic fuzzy topological space. Then the following statements are equivalent:

i) \((X, \tau)\) is an IFgsr cokernel compact space.

ii) For each IFgsrC-cocompact set \(A\), \(\text{IFgsrK}_c(A)\) is an IFgsrC-cocompact set.

iii) For each IFgsrC-cocompact set \(A\), we have \(\text{IFgsrCK}_c(\text{IFgsrCK}_c(A)) = \text{IFgsrCK}_c(A)\).

iv) For every pair of IFgsrC-cocompact sets \(A\) and \(B\) with \(\overline{\text{IFgsrCK}_c(A)} \subseteq \text{IFgsrCK}_c(B)\), we have \(\text{IFgsrCK}_c(A) = \text{IFgsrCK}_c(B)\).

**Proof:** (i) \(\Rightarrow\) (ii)

Let \(A\) be an IFgsrC-cocompact set in \((X, \tau)\) and \(\overline{A}\) is an IFgsrC-cocompact set in \((X, \tau)\). By assumption, we have \(\text{IFgsrCK}_c(\overline{A})\) is an IFgsrC-cocompact set in \((X, \tau)\). Now, \(\text{IFgsrCK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\). Therefore, \(\text{IFgsrK}_c(A)\) is an IFgsrC-cocompact set in \((X, \tau)\). Hence (i) \(\Rightarrow\) (ii).

(ii) \(\Rightarrow\) (iii)

Let \(A\) be an IFgsrC-cocompact set in \((X, \tau)\). Then \(\overline{A}\) is an IFgsrC-cocompact set in \((X, \tau)\). Given that \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\) is an IFgsrC-cocompact set.

Now, \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\). Hence (ii) \(\Rightarrow\) (iii).

(iii) \(\Rightarrow\) (iv)

Let \(A\) and \(B\) be any two IFgsrC-cocompact sets in \((X, \tau)\) such that (iii), \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\) which implies that \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\). Hence (iii) \(\Rightarrow\) (iv).

(iv) \(\Rightarrow\) (i)

Let \(A\) and \(B\) be any two IFgsrC-cocompact sets in \((X, \tau)\) such that \(B = \text{IFgsrK}_c(A)\). Given that, \(\text{IFgsrK}_c(A) = \text{IFgsrK}_c(A)\).

Also, \(\text{IFgsrK}_c(A) = \text{IFgsrK}_c(A)\) is an IFgsrC-cocompact set in \((X, \tau)\). This implies that \(\text{IFgsrK}_c(A)\) is an IFgsrC-cocompact set in \((X, \tau)\). Thus, \((X, \tau)\) is an IFgsrC-cocompact cokernel compact space. Hence (iv) \(\Rightarrow\) (i).

**Proposition 3.8:** Let \((X, \tau)\) be an intuitionistic fuzzy topological space. Then \((X, \tau)\) is an IFgsr cokernel compact space iff for each IFgsrC-cocompact set \(A\) and IFgsrC-cocompact set \(B\) such that \(A \subseteq B\), \(\text{IFgsrK}_c(A) \subseteq \text{IFgsrK}_c(B)\).

**Proof:** Let \((X, \tau)\) be an IFgsr cokernel compact space. Let \(A\) be an IFgsrC-cocompact set and \(B\) is an IFgsrC-cocompact set in \((X, \tau)\) such that \(A \subseteq B\).

Then by (ii) of proposition 3.7, \(\text{IFgsrK}_c(B)\) is an IFgsrC-cocompact set in \((X, \tau)\).

Therefore, \(\text{IFgsrK}_c(B) = \text{IFgsrK}_c(B)\). Since \(A\) is an IFgsrC-cocompact set and \(A \subseteq B\), \(A \subseteq \text{IFgsrK}_c(B)\).

Now, \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(\overline{A})\) and \(\text{IFgsrK}_c(B) = \text{IFgsrK}_c(B)\).

Conversely, let \(B\) be an IFgsrC-cocompact set in \((X, \tau)\), then \(\text{IFgsrK}_c(B)\) is an IFgsrC-cocompact set. By assumption, \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(B)\). Hence (i) of proposition \(\, (X, \tau)\) is an IFgsr cokernel compact space.

**Definition 3.9:** Let \((X, \tau)\) and \((Y, \delta)\) be any two IFgsr cokernel compact spaces. A function \(f : (X, \tau) \rightarrow (Y, \delta)\) is called an IFgsr \(C\)-compact open function if \(f(A)\) is an IFgsr C-cocompact set in \((Y, \delta)\) for each IFgsr C-cocompact set \(A\) in \((X, \tau)\).

**Proposition 3.10:** Let \((X, \tau)\) and \((Y, \delta)\) be any two IFgsr cokernel compact spaces. A function \(f : (X, \tau) \rightarrow (Y, \delta)\) be an IFgsr C-compact open and surjective function. Then for each \(f^{-1}(\text{IFgsrCK}_c(A)) \subseteq \text{IFgsrCK}_c(f^{-1}(A))\) intuitionistic set \(A\) in \((Y, \delta)\).

**Proof:** Let \(A\) be an intuitionistic fuzzy set in \((Y, \delta)\) and \(B = f^{-1}(A)\). Then, \(\text{IFgsrK}_c(\overline{A}) = \text{IFgsrK}_c(B)\) is an IFgsr C-cocompact set in \((X, \tau)\). Now, \(\text{IFgsrK}_c(B) \subseteq B\). Hence \(f(\text{IFgsrK}_c(B)) \subseteq f(B)\), i.e., \(\text{IFgsrK}_c(f(\text{IFgsrK}_c(B))) \subseteq \text{IFgsrK}_c(f(B))\).

Since \(f\) is an IFgsr C-compact open function, \(f(\text{IFgsrK}_c(B))\) is an IFgsrC-cocompact set in \((Y, \delta)\).

Therefore, \(f(\text{IFgsrK}_c(B)) \subseteq \text{IFgsrK}_c(f(B)) = \text{IFgsrK}_c(A)\).

Hence, \(\text{IFgsrK}_c(f^{-1}(\overline{A})) \subseteq f^{-1}(\text{IFgsrK}_c(B))\).

This implies that \(\text{IFgsrK}_c(\overline{f^{-1}(A)}) \subseteq f^{-1}(\text{IFgsrK}_c(B))\).

implies \(\text{IFgsrK}_c(f^{-1}(A)) \subseteq f^{-1}(\text{IFgsrK}_c(A))\). Hence the proof.

**Definition 3.11:** Let \((X, \tau)\) and \((Y, \delta)\) be any two IFgsr cokernel compact spaces. A function
f: (X, τ) → (Y, δ) is called an IFgsr C-compact continuous function if \( f^{-1}(A) \) is IFgsr C-compact set in (X, τ) for every IFgsr C-compact set A in (Y, δ).

**Remark 3.12:** Let (X, τ) and (Y, δ) be any two IFgsr kernel compact spaces. Let \( f: (X, \tau) \to (Y, \delta) \) be any function. Then the following statements are equivalent:

(i) \( f: (X, \tau) \to (Y, \delta) \) is an IFgsr C-compact continuous function.

(ii) IFgsrCKc_\sim (f^{-1}(A)) \subseteq f^{-1}(IFgsrCKc_\sim (A)) \text{ for each } IFgsr C-compact set A in (Y, \delta).

**Proof:** (i)⇒(ii)

Given \( f: (X, \tau) \to (Y, \delta) \) is an IFgsr C-compact continuous function. Let \( A \subseteq (Y, \delta) \) be any IFgsr C-compact set in (Y, \delta). Let \( f^{-1}(A) \) be an Igsr C-compact set in (X, τ) and hence \( f^{-1}(IFgsrCKc_\sim (A)) \) is an IFgsr C-compact set in (X, τ). Therefore, \( IFgsrCKc_\sim (f^{-1}(IFgsrCKc_\sim (A))) = f^{-1}(IFgsrCKc_\sim (A)) \) since \( f^{-1}(A) \subseteq (Y, \delta) \). Thus, \( f^{-1}(f^{-1}(IFgsrCKc_\sim (A))) = f^{-1}(IFgsrCKc_\sim (A)) \).

(iii)⇒(ii)

Given that \( IFgsrCKc_\sim (f^{-1}(A)) \subseteq f^{-1}(IFgsrCKc_\sim (A)) \), for each IFgsr C-compact set A in (Y, δ). Let A be an IFgsr C-cocompact set in (Y, δ). It is enough to show that \( f^{-1}(A) \) is an IFgsr C-compact set in (X, τ). Since A = IFgsrCKc_\sim (A), \( f^{-1}(A) = f^{-1}(IFgsrCKc_\sim (A)) \) but it is given that \( f^{-1}(A) \subseteq (Y, \delta) \). Hence \( f^{-1}(A) \subseteq (Y, \delta) \). Thus \( f^{-1}(A) = f^{-1}(IFgsrCKc_\sim (A)) \). i.e., \( f^{-1}(A) \) is an IFgsr C-compact set in (X, τ). This proves that \( f^{-1}(A) \) is an IFgsr C-compact function.

**Proposition 3.13:** Let (X, τ) and (Y, δ) be any two IFgsr kernel compact spaces. Let \( f: (X, \tau) \to (Y, \delta) \) be a bijective function. Then f is an IFgsr C-compact continuous function if for every intuitionistic fuzzy set A in (X, τ), \( f(\text{IFgsrCKc}_\sim (A)) \subseteq \text{IFgsrCKc}_\sim (f(A)) \).

**Proof:** Let us assume that f is an IFgsr C-compact continuous function and A is an intuitionistic fuzzy set in (X, τ). Hence, \( f^{-1}(\text{IFgsrCKc}_\sim (A)) \) is an IFgsr C-compact set in (X, τ). By Remark 3.12, \( f^{-1}(f(A)) \subseteq f^{-1}(\text{IFgsrCKc}_\sim (f(A))) \). Taking both sides, \( f(\text{IFgsrCKc}_\sim (A)) \subseteq f^{-1}(\text{IFgsrCKc}_\sim (f(A))) \). Since f is a surjective function, \( f(\text{IFgsrCKc}_\sim (A)) \subseteq \text{IFgsrCKc}_\sim (f(A)) \).

**Proposition 3.14:** Let (X, τ) and (Y, δ) be any two IFgsr kernel compact spaces. Let \( f: (X, \tau) \to (Y, \delta) \) be any function. Then the following statements are equivalent:

(i) \( f: (X, \tau) \to (Y, \delta) \) is an IFgsr C-compact continuous function.

(ii) \( \text{IFgsrCKc}_\sim (f(A)) \subseteq f(\text{IFgsrCKc}_\sim (A)) \) for each IFgsr C-compact set A = \( \mu_A(x), \nu_A(x) > 0 \) in (X, τ).

**Proof:** (i)⇒(ii)

Let \( A = \mu_A(x), \nu_A(x) > 0 \) be an IFgsr C-compact set in (X, τ). Clearly \( \text{IFgsrCKc}_\sim (A) \) is an IFgsr C-compact set in (X, τ). Since f is an IFgsr C-compact function, \( f(\text{IFgsrCKc}_\sim (A)) \) is an IFgsr C-cocompact set in (Y, δ).

Thus \( f(\text{IFgsrCKc}_\sim (f(A))) \subseteq f(\text{IFgsrCKc}_\sim (A)) \).

Hence (i)⇒(ii).

(ii)⇒(i)

Let A be any IFgsr C-cocompact set in (X, τ). Then \( A = \text{IFgsrCKc}_\sim (A) \). By (ii), \( f(\text{IFgsrCKc}_\sim (f(A))) \subseteq f(A) \). Thus \( f(A) = \text{IFgsrCKc}_\sim (f(A)) \) and hence \( f(A) \) is an IFgsr C-cocompact set in (Y, δ). Therefore, f is an IFgsr C-cocompact function. Hence (ii)⇒(i).

**Definition 3.15:** Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. A function \( f: (X, \tau) \to (Y, \delta) \) is called an IFgsr C-compact irresolute function if \( f^{-1}(A) \) is IFgsr C-compact set in (X, τ) for each IFgsr C-compact set A in (Y, δ).

**Proposition 3.16:** Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. A function \( f: (X, \tau) \to (Y, \delta) \) is an IFgsr C-compact irresolute function if and only if \( f(\text{IFgsrCKc}_\sim (A)) \subseteq \text{IFgsrCKc}_\sim (f(A)) \) for every IFgsr C-compact set A in (X, τ).
Proof: Suppose that $f$ is an IFgsr C-compact irresolute function and let $A$ be an IFgsr C-compact set in $(X, \tau)$. Then, $\text{IFgsrCK}_C^\delta(f(A))$ is an IFgsr C-cocompact set in $(Y, \delta)$. By assumption, $f^{-1}(\text{IFgsrCK}_C^\delta(f(A)))$ is an IFgsr C-compact set in $(X, \tau)$. Now, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\text{IFgsrCK}_C^\delta(f(A)))$. Now, $A \subseteq f^{-1}(\text{IFgsrCK}_C^\delta(f(A)))$.

Remark 4.3: Every IFgsr RC-compact is an IFgsr C-compact.

Proposition 4.4: Let $(X, \tau)$ and $(Y, \delta)$ be any two intuitionistic fuzzy topological spaces. A function $f: (X, \tau) \rightarrow (Y, \delta)$ is an IFgsr C-compact continuous function of $(X, \tau)$ into an IFgsr cokernel compact space $(Y, \delta)$ if $V = \mu(x)$ and $V(x) > 0$ is an IFgsr RC-compact in $(Y, \delta)$, then $f^{-1}(V)$ is an IFgsr RC-compact in $(X, \tau)$.

4. PROPERTIES OF IFGSR R-COMPACT SPACES

Definition 4.1: Let $(X, \tau)$ be an IFgsr cokernel compact space and let $A = \mu(x)$, $\nu(x) > 0$ be any intuitionistic fuzzy set in $(X, \tau)$. Then $A$ is said to be an IFgsr R-compact if $A = \text{IFgsrKR}_C^\delta(\text{IFgsrKR}_C^\delta(A)).$

Definition 4.2: Let $(X, \tau)$ be an IFgsr cokernel compact space and let $A = \mu(x)$, $\nu(x) > 0$ be any intuitionistic fuzzy set in $(X, \tau)$. Then $A$ is said to be an IFgsr RC-compact if $A = \text{IFgsrKR}_C^\delta(\text{IFgsrKR}_C^\delta(A)).$

Remark 4.4: Every IFgsr RC-compact is an IFgsr C-compact.
Since f is an IFgsr C-compact bijective function, f is an IFgsr C-cocompact function. Hence, IFgsrK_{c}(f(V)) \subseteq f(\text{IFgsrK}_{c}(V)) \Rightarrow f(V) \text{then,}\n\text{IFgsrK}_{c}(f(V)) \subseteq f(V) \quad \text{-----------7}

From 6 and 7, it follows that IFgsrK_{c}(f(V)) = f(V) i.e., f(V) is an IFgsr RC-compact set in (Y, δ).

**Definition 4.8:** Let (X, τ) be an intuitionistic fuzzy topological space. If a family \{G_{i} = < x, \mu_{G_{i}}(x), \nu_{G_{i}}(x) ; i \in J \} of IFgsr RC-compact (X, τ) satisfies the condition \bigcup \{G_{i} ; i \in J \} = X, then it is called IFgsr RC-compact cover of X.

**Definition 4.9:** An intuitionistic fuzzy topological space (X, τ) is said to be IFgsr RC-compact space if and only if every IFgsr RC-cover of (X, τ) has a finite subfamily, the IFgsr C-compact cokernels of whose members cover the space (X, τ).

**Proposition 4.10:** Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. Let f: (X, τ) \rightarrow (Y, δ) be an IFgsr C-compact function of an IFgsr RC-compact space (X, τ) onto an IFgsr cokernel compact space (Y, δ), then (Y, δ) is an IFgsr R-compact space.

**Proof:** Let V_{j} = < x, \mu_{V_{j}}(x), \nu_{V_{j}}(x) > be an IFgsr RC-compact cover of (Y, δ). Since f is an IFgsr C-compact continuous function and \{f^{-1}(V_{j}) : j \in J \} is an IFgsr cokernel compact space, from Proposition 4.4, \{f^{-1}(V_{j}) : j \in J \} is an IFgsr RC-compact cover of (X, τ). Since (X, τ) is an IFgsr R-compact space, there is a finite subfamily such that \bigcup_{i=1}^{n} f^{-1}(V_{j_{i}}), \ldots, f^{-1}(V_{j_{n}}) \text{ such that } \tilde{X} = \bigcup_{i=1}^{n} \text{IFgsrK}_{c}(f^{-1}(V_{j_{i}})).

Thus, \tilde{Y} = f(\bigcup_{i=1}^{n} \text{IFgsrK}_{c}(f^{-1}(V_{j_{i}}))) \subseteq \bigcup_{i=1}^{n} \text{IFgsrK}_{c}(V_{j_{i}}).

Hence \tilde{Y} = \bigcup_{i=1}^{n} \text{IFgsrK}_{c}(V_{j_{i}}).

**Proposition 4.11:** Let (X, τ) and (Y, δ) be any two intuitionistic fuzzy topological spaces. Let f: (X, τ) \rightarrow (Y, δ) be an IFgsr C-compact continuous bijective function of an IFgsr cokernel compact space (X, τ) onto an IFgsrRC-compact space (Y, δ), then (X, τ) is an IFgsr C-compact space.

**Proof:** Let V_{a} = < x, \mu_{V_{a}}(x), \nu_{V_{a}}(x) > be an IFgsr RC-compact cover of (X, τ). From Proposition 4.7, \{f^{-1}(V_{a}) : j \in J \} is an IFgsr RC-compact cover of (Y, δ).

Since (Y, δ) is an IFgsr R-compact space, there is a finite subfamily such that \bigcup_{i=1}^{n} f^{-1}(V_{a_{c}}), \ldots, f^{-1}(V_{a_{n}}) \text{ such that } \tilde{Y} = \bigcup_{i=1}^{n} \text{IFgsrK}_{c}(f^{-1}(V_{a_{i}})).

Thus, \tilde{X} = f^{-1}(\bigcup_{i=1}^{n} \text{IFgsrK}_{c}(f^{-1}(V_{a_{i}}))) = f^{-1}(\bigcup_{i=1}^{n} \text{IFgsrK}_{c}(V_{a_{i}})).

Therefore, (X, τ) is an IFgsr C-compact space.

**References**


